Computational Toric Geometry
Géométrie torique computationelle

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TRISTRAM BOGART, MSRI/San Francisco State University

Few smooth d-polytopes with N lattice points

The AIM workshop "Combinatorial Challenges in Toric Varieties" centered on several questions about lattice polytopes motivated by toric geometry. In particular, a lattice polytope is said to be smooth if it is simple and the primitive rays of each vertex cone comprise a lattice basis. The questions are whether smoothness implies any or all of a list of successively stronger properties including normality, Koszul, and existence of a regular unimodular triangulation.

To gather evidence for these questions, we set out to test the properties systematically in dimension three. In order to do this, we proved a key finiteness property: given a finite list F of rational cones and an integer N, there are only finitely many lattice polytopes whose normal cones are (integrally equivalent to) elements of F and that contain at most N lattice points. In particular this holds if F simply consists of the unimodular cone, the smooth case. The proof is algorithmic and was implemented by Benjamin Lorenz to perform computations that give a positive answer to all of the questions for the case of three-polytopes with at most twelve lattice points. In algebro-geometric language, the strongest result is that if X is a toric three-fold embedded in $\mathbb{P}^{N-1}$ by a complete linear series for $N \leq 12$, then the defining ideal $I(X)$ has a square-free quadratic initial ideal.

This project is joint work with Christian Haase, Milena Hering, Lorenz, Benjamin Nill, Andreas Paffenholz, Francisco Santos, and Hal Schenck.

VINCENT BOUCHARD, University of Alberta

Tops

It is well known that reflexive polytopes can be used to study Calabi-Yau hypersurfaces in toric varieties. Less however is known about the use of "tops" — three-dimensional lattice polytopes with one facet containing the origin and the other facets at integral distance one from the origin. Those appear in the context of elliptically fibered Calabi-Yau hypersurfaces, where they encode degenerations of elliptic fibers. In this talk I will summarize the uses of tops, and explain how they can be classified into families closely related to the classification of affine Kac-Moody algebras.

VOLKER BRAUN, Dublin Institute for Advanced Studies

Toric Geometry in Sage

In this talk, I will present the toric geometry package that I developed jointly with Andrey Novoseltsev. It is implemented in the Sage computer mathematics system and provides an efficient and user-friendly implementation of many toric algorithms. As an application, I will construct an elliptically fibered fourfold whose discriminant locus contains a non-simply connected divisor. This is relevant for F-theory compactifications of string theory.

CHARLES DORAN, Alberta

ANDREY NOVOSELTSEV, University of Alberta

Geometric transitions through singular subfamilies in toric varieties

(Joined work with Charles F. Doran.) We present an explicit geometric transition between Calabi-Yau threefolds realized as anticanonical hypersurfaces and complete intersections in toric varieties, where the role of the singular variety in the geometric transition is played by a generically singular subfamily of hypersurfaces. The involved families of Calabi-Yau varieties may lead to a representative for the last class in Doran-Morgan classification of variations of Hodge structure associated to one-parameter families of Calabi-Yau threefolds. In the exploration of this example we have extensively used the newly developed framework for toric geometry in Sage.

URSULA WHITCHER, Harvey Mudd College
The Griffiths-Dwork algorithm for toric hypersurfaces

The variation of complex structure for a family of Calabi-Yau varieties is encoded in a differential equation called the Picard-Fuchs equation. We develop computational methods for studying properties of Calabi-Yau varieties realized as hypersurfaces in toric varieties, and implement an algorithm for computing their Picard-Fuchs equations. We apply our methods to study highly symmetric families of K3 surfaces. This talk describes joint work with Dagan Karp, Jacob Lewis, Daniel Moore, and Dmitri Skjorshammer.