Applicable Harmonic Analysis and Approximation Theory
Analyse harmonique appliquée et théorie d'approximation

(Org: **Bin Han** (Alberta))

TWAREQUE ALI, Concordia University, Montreal

The Galilei group and its extensions in constructing signal transforms

We show how certain extensions of the Galilei group in (1+1)-dimensions (in space-time), which are physical kinematic groups, accomodate all the groups currently used in constructing signal transforms, viz, the affine, the Weyl-Heisenberg, the shearlet and the Stockwell groups. We also analyze how the signal transforms themselves sit within representations of the larger group. The results demonstrate a remarkable unification of the various signal transforms currently used in the literature.

ELENA BRAVERMAN, University of Calgary

On fast multiscale algorithms incorporating FFT and domain decomposition

We present fast spectral algorithms for the solution of elliptic partial differential equations (the Poisson equation, the Helmholtz and the modified Helmholtz equations) which may involve decomposition of the original domain. In each subdomain, the solution is based on the application of the Fast Fourier Transform. The ways to handle the Gibbs phenomen (including 2-D and 3-D domains) are discussed. One of the methods is based on the eigenfunction expansion of the right hand side with successive integration combined with a subtractional technique.

FENG DAI, University of Alberta

weighted quadrature in Besov spaces with A_∞ weights on multivariate domains

Let Ω denote either the unit sphere $S^{d-1} := \{x \in \mathbb{R}^d : \ \|x\| = 1\}$, or the unit ball $B^d := \{x \in \mathbb{R}^d : \ \|x\| \le 1\}$, or the standard simplex $T^d := \{x \in \mathbb{R}^d : \ \ x_1, \cdots, x_d \ge 0, \sum_{j=1}^d x_j \le 1\}$, and let w be an A_∞ weight on Ω. For the unit ball $MB^\alpha_\tau(L_{p,w})$ of the weighted Besov space $B^\alpha_\tau(L_{p,w})$ on Ω, we find the sharp asymptotic order of the following quantity as $n \to \infty$:

$$\inf_{\substack{\lambda_1,\cdots,\lambda_n \in \mathbb{R} \\ \xi_1,\cdots,\xi_n \in \Omega}} \sup_{f \in MB^\alpha_\tau(L_{p,w})} \Big| \int_\Omega f(x) w(x)\, dx - \sum_{j=1}^n \lambda_j f(\xi_j) \Big|.$$

We also establish a similar result on unweighted spherical caps.

ZEEV DITZIAN, University of Alberta

Use of Convexity of the Unit Ball of a Space for Improving on the Jackson Inequality

The Littlewood-Paley inequality has been used to sharpen the Jackson inequality for L_p when $p \in (1, \infty)$ and $s = \max(p, 2)$.These new inequality was an improvement on the Jackson inequality which yield better asymptotic behavior. A recent different technique uses conditions on the convexity of the unit ball of a space B related to s to achieve such results which we call sharp Jackson inequalities. This allow us to obtain Sharp Jackson inequalities for spaces and systems of orthogonal expansions for which the Littlewood -Paley inequality cannot be employed.

MIROSLAV ENGLIS, Institute of Mathematics, Academy of Sciences, Czech Republic

Wigner transform on symmetric spaces

Extending previous work by T. Tate, M. Bertola, and others, we present a generalization of the Wigner transform and Weyl pseudo-differential operators from the Euclidean context to symmetric spaces of non-compact type.

(Joint work with S.-T. Ali, Montreal.)

JEAN-PIERRE GABARDO, McMaster University
Self-affine scaling sets in $\mathbb{R}^2$*.*

Let A be an $n \times n$ integral expansive matrix with $|\det A| = 2$. A measurable set K is called an A-dilation scaling set (resp. MRA scaling set) if $Q = BK \setminus K$ is an A-dilation wavelet set (resp. MRA wavelet set), where $B = A^t$. In this talk, we give a complete characterization of all two dimensional A-dilation scaling sets K such that K is at the same time a self-affine tile associated with B, i.e. K satisfies $BK = K \bigcup (K + d)$ for some $d \in \mathbb{R}^2$. In addition, we show that all such scaling sets must be MRA scaling sets.This is joint work with Xiaoye Fu.

BIN HAN, University of Alberta
The Matrix Extension Problem and Orthogonal Wavelets over Algebraic Number Fields

As a finite dimensional linear space over the rational number field, an algebraic number field is of particular interest in mathematics and engineering. Algorithms using algebraic number fields can be efficiently implemented involving only integer arithmetics. We observe that all finitely supported orthogonal low-pass (multi)wavelet filters known in the literature have coefficients from an algebraic number field. Therefore, it is of interest to study orthogonal (multi)wavelet filter banks over algebraic number fields. In this talk, we shall formulate the matrix extension problem over a general subfield of the complex number field (including algebraic number fields as special cases). The core task of the matrix extension problem is to extend a given $r \times s$ matrix, with $1 \leq r \leq s$, of Laurent polynomials into an $s \times s$ square paraunitary matrix with some desirable properties such as symmetry and short supports. We shall provide a complete satisfactory solution to this matrix extension problem and discuss its applications to symmetric orthogonal (multi)wavelets. Several examples of symmetric real-valued or complex-valued orthogonal wavelets are provided to illustrate the results. This is joint work with Xiaosheng Zhuang. Preprints are available at http://www.ualberta.ca/∼bhan

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KIRILL KOPOTUN, University of Manitoba
Q-monotone spline smoothing

Several results on q-monotone spline smoothing will be discussed (this is a joint work with D. Leviatan and A. Prymak). In particular, we show how one can constructively smooth any monotone or convex piecewise polynomial function (ppf) (or any q-monotone ppf, $q \geq 3$, with one additional degree of smoothness) to be of minimal defect while keeping it close to the original function in the L_p-(quasi)norm.

One of the applications of this smoothing will be discussed in this session by A. Prymak in his talk "Three-monotone spline approximation".

MING-JUN LAI, University of Georgia
Recent Advances on Sparse Solution and its Application for Image Denoising

We shall first survey recent developments on the ℓ_q minimization approach with $q < 1$ for the sparse solution of under-determined linear systems. This approach includes both constrained and unconstrained ℓ_q minimizations.

Next we shall explain recent developments on greedy algorithms for the sparse solution. Mainly, we shall provide a convergence analysis of an iteratively least squares orthogonal greedy algorithm. We use this algorithm for image denoising.

Finally, we present some numerical evidence of image and movie denoising.

ANDRIY PRYMAK, University of Manitoba
Three-monotone spline approximation

For $r \geq 3$, $n \in \mathbb{N}$ and each 3-monotone continuous function f on $[a,b]$ (i.e., f is such that its divided differences $[x_0,x_1,x_2,x_3]f \geq 0$ for any distinct points $x_0,\ldots,x_3 \in [a,b]$), we construct a spline s of degree r and of minimal defect ($s \in C^{r-1}[a,b]$) with $n-1$ equidistant knots in (a,b), which is also 3-monotone and satisfies

$$\|f-s\|_{L_\infty[a,b]} \leq c\omega_4(f,n^{-1},[a,b])_\infty,$$

where $\omega_4(f,t,[a,b])_\infty$ is the 4-th modulus of smoothness of f in the uniform norm. This establishes the only remaining unproved Jackson type estimate for uniform 3-monotone approximation by splines with uniformly spaced fixed knots.

First we prove this estimate for s with the knots that are allowed to depend on f but cannot be too close to each other ("controlled" knots). Then we use very recent results on constrained spline smoothing to achieve maximum smoothness and to move the knots to the right place.

Moreover, we also prove a similar estimate in terms of the Ditzian-Totik 4-th modulus of smoothness for splines with Chebyshev knots, and show that these estimates are no longer valid in the case of 3-monotone spline approximation in the L_p norm with $p < \infty$. At the same time, positive results in the L_p-case with $p < \infty$ are still valid for splines with "controlled" knots.

These results confirm that 3-monotone approximation is the transition case between monotone and convex approximation (where most of the results are "positive") and k-monotone approximation with $k \geq 4$ (where just about everything is "negative").

KEITH TAYLOR, Dalhousie University
Variations on the Shearlet Transform

The continuous shearlet transform is applicable to functions of two real variables such as those representing images. It is effective when there are one dimensional singularities in various orientations. This talk will place the shearlet transform in the context of a larger family of transforms arising from adding dilations to the Heisenberg group.

OZGUR YILMAZ, University of British Columbia
Compressed sensing with partial support information

Compressed sensing is a powerful "non-adaptive" signal acquisition paradigm. After making the initial assumption that the high-dimensional signals to be acquired are sparse or compressible, one constructs a universal sampling method (i.e., a measurement matrix) that will provide sufficient information to recover exactly or approximately the underlying signal. Typically, the reconstruction algorithm (e.g., ℓ_1-minimization) is also non-adaptive—i.e., it does not utilize any additional information about the signal and can be used to estimate any sufficiently sparse or compressible signal. In various applications, however, there is prior information that can be exploited to improve the recovery quality (this is the basis of various approaches that fall under the term "model-based compressed sensing"). In this talk, we present such a method that improves signal reconstruction from compressed sensing measurements when partial support information is available. We propose to use a certain weighted ℓ_1-minimization algorithm in this setting. We prove that if at least 50% of the (partial) support information is accurate, then weighted ℓ_1-minimization is stable and robust under weaker conditions than those for standard ℓ_1 minimization. Moreover, weighted ℓ_1-minimization provides better bounds on the reconstruction error in terms of the measurement noise and the compressibility of the signal to be recovered. We illustrate our results with extensive numerical experiments on synthetic data and real audio and video signals. We also propose an iterative algorithm that is based on a "support estimate" stage followed by a weighted ℓ_1-minimization algorithm. This is joint work with Hassan Mansour, Rayan Saab, and Michael Friedlander.