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Three-monotone spline approximation

For $r \geq 3$, $n \in \mathbb{N}$ and each 3-monotone continuous function f on $[a,b]$ (i.e., f is such that its divided differences $[x_0, x_1, x_2, x_3]f \geq 0$ for any distinct points $x_0, \ldots, x_3 \in [a,b]$), we construct a spline s of degree r and of minimal defect ($s \in C^{r-1}[a,b]$) with $n-1$ equidistant knots in (a,b), which is also 3-monotone and satisfies

$$\|f - s\|_{L_\infty[a,b]} \leq c\omega_4(f, n^{-1}, [a,b])_\infty,$$

where $\omega_4(f,t,[a,b])_\infty$ is the 4-th modulus of smoothness of f in the uniform norm. This establishes the only remaining unproved Jackson type estimate for uniform 3-monotone approximation by splines with uniformly spaced fixed knots.

First we prove this estimate for s with the knots that are allowed to depend on f but cannot be too close to each other ("controlled" knots). Then we use very recent results on constrained spline smoothing to achieve maximum smoothness and to move the knots to the right place.

Moreover, we also prove a similar estimate in terms of the Ditzian-Totik 4-th modulus of smoothness for splines with Chebyshev knots, and show that these estimates are no longer valid in the case of 3-monotone spline approximation in the L_p norm with $p < \infty$. At the same time, positive results in the L_p-case with $p < \infty$ are still valid for splines with "controlled" knots.

These results confirm that 3-monotone approximation is the transition case between monotone and convex approximation (where most of the results are "positive") and k-monotone approximation with $k \geq 4$ (where just about everything is "negative").