Tensor Categories
Catégories tensorielles
(Org: **Robert Paré** (Dalhousie))

MARGARET BEATTIE, Mount Allison University
Cocycle twists of bialgebras in a Yetter–Drinfel'd category

Let H be a Hopf algebra over a field k. In this talk we discuss bialgebras in the category ${}^H_H\mathcal{YD}$ of left-left Yetter–Drinfel'd modules over H. These are of interest in the following problem. Define a splitting datum to be a 4-tuple (A, H, π, σ) where A is a bialgebra, $\sigma\colon H \to A$ is a bialgebra map and $\pi\colon A \to H$ is an H-bilinear coalgebra map such that $\pi\sigma = \mathrm{Id}_H$. The set of coinvariants R of π is a coaugmented coalgebra in the category ${}^H_H\mathcal{YD}$ but is not, in general, a bialgebra unless the projection π is also an algebra map. In any case, we can write $A \cong R\#_\xi H$, a modification of a Radford biproduct, with ξ trivial when π is an algebra map. We ask the following question: If ξ is not trivial, can we twist A by a cocycle so that $A^\gamma \cong R'\#H$ where R' is a bialgebra in ${}^H_H\mathcal{YD}$, i.e., can we twist A so that the twist is isomorphic to a Radford biproduct?

MARTA BUNGE, McGill University, Department of Mathematics and Statistics, 805 Sherbrooke St West, Montreal, QC H3A 2K6
A characterization of frames as suplattices without the use of the tensor product

A characterization of frames in terms of suplattices is given in [1] as certain commutative monoids. It is not internal to the tensored category of suplattices in that the diagonal map, which is not part of the structure, is employed. It is used therein in the proof of their main theorem, namely, that open surjections of locales and toposes are of effective descent.

I will give an alternative characterization of frames that is internal to the category $\mathbf{sl}$ of suplattices and sup-preserving maps—in particular, without the use of the tensor product. The key ingredient is a construction [3] of the lower power locale along the lines of that of the symmetric topos [2]. I shall argue that this alternative characterization can equally well be employed in the proof of the main descent theorem of [1].

The entire discussion is done relative to an arbitrary base topos $\mathcal{S}$.

References

[1] A. Joyal and M. Tierney, *An extension of the Galois theory of Grothendieck*. Mem. Amer. Math. Soc. **309**, 1984.

[2] M. Bunge and A. Carboni, *The symmetric topos*. J. Pure Appl. Algebra **105**(1995), 233–249.

[3] M. Bunge and J. Funk, *Constructive theory of the lower power locale*. Math. Str. Comp. Science **6**(1996), 1–15.

GEOFF CRUTTWELL, University of Calgary, 2500 University Drive NW, Calgary, AB
Generalized multicategories

The notion of a generalized multicategory has been defined in a number of different contexts by Hermida, Clementino/Tholen, Leinster, and others. It includes such diverse examples as topological spaces, symmetric multicategories, and Lawvere theories. In each case, the author works with a "monad-like" functor on a bicategory, and shows that its "algebras" are generalized multicategories.

We will discuss a framework for generalized multicategories which uses monads on double categories (rather than on bicategories). By moving to this level of generality, we can unify all previous examples, while at the same time showing that definitions such as functors between generalized multicategories have a natural interpretation.

This is joint work with Mike Shulman.

JEFF EGGER, University of Edinburgh
A non-"evil" definition of dagger compact closed category?

In this talk, I will give a definition of *cyclic involutive monoidal category* and of *coherent family of Hermitian adjunctions* such that a cyclic (respectively: balanced, symmetric) involutive monoidal category together with a chosen coherent family of Hermitian adjunctions is "the same thing as" a dagger pivotal (respectively: tortile, compact closed) category.

PETER FREYD, University of Pennsylvania

PIETER HOFSTRA, University of Ottawa, 585 King Edward Ave., Ottawa, ON, K1N 6N5
Actions of semigroups and groupoids

This talk explores the interplay between topos theory, inverse semigroup theory and continuous groupoids, focusing on categorical interpretations of various kinds of (continuous) actions of inverse semigroups.

Based on joint work with Jonathon Funk and Benjamin Steinberg.

TOBY KENNEY, Dalhousie University, Halifax, NS
Categories as Monoids in Span, Rel, and Sup

We look at various internal constructions in various categories and bicategories, that are equivalent to categories. In particular, categories can be expressed as special types of monoids in the category $\mathcal{S}pan$, whose objects are sets, and whose morphisms are spans of functions. In fact, these monoids also live in the category $\mathcal{R}el$, of sets and relations. There is a well-known equivalence between $\mathcal{R}el$, and a full subcategory of the category $\mathcal{S}up$, of complete lattices and sup-preserving morphisms. This allows us to represent categories as a special kind of monoid in $\mathcal{S}up$. Monoids in $\mathcal{S}up$ are called quantales, and are of interest in a number of different areas.

We will also study the appropriate ways to express other categorical structures such as functors, natural transformations and profunctors in these categories.

Joint work with R. Paré.

FRED LINTON, Wesleyan Univ., 125 Science Tower, Middletown, CT 06459, USA
Meditations on Arens Multiplication

Between the years 1945, when Eilenberg and MacLane officially made public the notions of category, functor, natural transformation, and natural equivalence, and 1965, when Eilenberg and Kelly released their definitive opus on closed and monoidal categories, there were other tentative steps towards a realization of the concept of closed category, steps focussing, for the most part, on Banach spaces or more general sorts of spaces of interest to Analysts: there were the 1960-vintage Doklady and Uspechi articles of Fuks, Shvarts, and Mityagin on duality of functors on Banach spaces; the 1955 work of Grothendieck on topological tensor products; Robert Schatten's 1950 Annals Study, *A Theory of Cross-Spaces*; and (the focus of this presentation) the 1950 work of Richard Arens (published in 1951) on what has come to be called the Arens multiplication on the second conjugate A^{**} of a Banach algebra A, a construction relying, conceptually, on what Arens there called *phyla*, entities which correspond, grosso modo, to a kind of blend of what are known today as (associative) closed monoidal categories and today's multilinear categories.

The aim of this presentation is to publicize more widely this 60-year-old work of Arens that so unexpectedly pre-figured today's closed monoidal categories, and to bring into relief some of the as yet unsolved problems it both posed then and suggests today.

References

[1] Samuel Eilenberg and Saunders MacLane, *General Theory of Natural Equivalences.* Trans. Amer. Math. Soc. **58**(1945), 231–294.

[2] Richard Arens, *Operations induced in function classes.* Monatsh. Math. **55**(1951), 1–19.

MICHAEL MAKKAI, McGill University, Montreal, Quebec
Weakly closed structures on higher-dimensional categories

In this talk, I return to the point of view of Eilenberg's and Kelly's "Closed Categories", which considers the closed structure, based on hom-objects, as primary, and the tensor product as secondary. The motivating example is Gray, the 3-dimensional category that can be defined as a closed category whose objects are (small) 2-categories, and whose arrows are 2-functors, by specifying the hom-objects in a relatively simple way (involving pseudo-natural transformations), without mentioning the (in fact, available) symmetric monoidal structure. There are several other examples of categories, with objects certain kinds of higher-dimensional categories, which have a natural candidate for a concept of internal hom, and which turn out to be closed categories, in a weakened sense at least. One is Gray-Cat, whose objects are Gray-categories. In a paper from 1999, Sjoerd Crans defines a tensor product on Gray-Cat, of which he shows that it does not carry a closed structure. My approach is of the opposite kind to Crans's in starting with a (weakly) closed structure that is not only simple but also useful in developing 2-dimensional category theory, for instance 2-dimensional Gabriel–Ulmer duality. Another example has objects tricategories. This last example has been used to give a proof of a strong form of the coherence theorem for tricategories.

OCTAVIO MALHERBE, University of Ottawa
A categorical model of higher-order quantum computation

Higher-order quantum computation, in the sense of Valiron and Selinger, is a language based on the lambda-calculus and linear logic. In this presentation I will describe how to build some concrete models of such language using techniques from tensor category theory.

MICAH MCCURDY, Macquarie University, North Ryde, New South Wales 2109, Australia
String and Stripes: Graphical Notation for Functors between Tensor Categories

Building on the "functorial boxes" introduced by Cockett and Seely and recently revived by Mellies, we introduce a notation for functors between tensor categories based on an intuition of "piping". This notation extends smoothly to encompass natural transformations, and is especially strong in showing naturality. I will give some examples of existing proofs rewritten in this notation to show its elegance, such as the well-known fact that a monoidal natural transformation between two monoidal functors with rigid domain is necessarily invertible. Time permitting, I will also discuss extensions to cover monoidal monads and other structures.

SUSAN NIEFIELD, Union College
The Glueing Construction and Double Categories

For a small category $\mathbf{B}$ and a double category $\mathbb{D}$, let $\mathrm{Lax}_N(\mathbf{B},\mathbb{D})$ denote the category whose objects are vertical normal lax functors $\mathbf{B} \to \mathbb{D}$ and morphisms are horizontal lax transformations. If $\mathbb{D}$ is the double category of toposes, locales, or topological spaces (see table below), then the glueing construction induces a functor from $\mathrm{Lax}_N(\mathbf{B},\mathbb{D})$ to the horizontal category $\mathbf{H}\mathbb{D}$.

Objects	Horizontal 1-Cells	Vertical 1-Cells	2-Cells
toposes $\mathcal{X}$	geometric morphisms $\mathcal{X} \to \mathcal{B}$	finite limit preserving $\mathcal{X}_1 \nrightarrow \mathcal{X}_2$	$\begin{array}{ccc} \mathcal{X}_1 & \to & \mathcal{B}_1 \\ \downarrow & \leftarrow & \downarrow \\ \mathcal{X}_2 & \to & \mathcal{B}_2 \end{array}$
locales X	locale morphisms $X \to B$	finite meet preserving $X_1 \nrightarrow X_2$	$\begin{array}{ccc} X_1 & \to & B_1 \\ \downarrow & \geq & \downarrow \\ X_2 & \to & B_2 \end{array}$
spaces X	continuous functions $X \to B$	finite meet preserving $\mathcal{O}(X_1) \nrightarrow \mathcal{O}(X_2)$	$\begin{array}{ccc} \mathcal{O}(X_1) & \to & \mathcal{O}(B_1) \\ \downarrow & \geq & \downarrow \\ \mathcal{O}(X_2) & \to & \mathcal{O}(B_2) \end{array}$

For each of these double categories, we know that $\mathrm{Lax}_N(\mathbf{2}, \mathbb{D})$ is equivalent to $\mathbf{H}\mathbb{D}/S$, where $\mathbf{2}$ is the 2-element totally ordered set and S is the Sierpinski object of $\mathbb{D}$. In this talk, we consider analogues of this equivalence for more general categories $\mathbf{B}$.

BOB PARÉ, Dalhousie University, Halifax, NS B3H 3J5
Mealy morphisms of enriched categories

The realization by Walters that enriched categories can be viewed as lax morphisms from an indiscrete category to a bicategory with one object has led to a new kind of morphism of enriched category, intermediate between strong functor and profunctor. We call these Mealy morphisms. They are a two-fold generalization of Mealy machines introduced in the 1950s in theoretical computer science. We will introduce the notion and examine some of its properties and give examples.

DORETTE PRONK, Department of Mathematics and Statistics, Dalhousie University, Halifax, NS B3H 3J5
Adjoining Adjoints to a Free Category on a Graph

The Π_2-construction, introduced in [1], freely adds right adjoints to the arrows of a category. When this construction is applied to a category that is freely generated on a graph it produces a 2-category where the arrows are paths of forward and backward arrows in the graph, and the 2-cells are Kauffman diagrams, where the strings are directed and labeled by the arrows of the graph.

This construction can be extended to 2-graphs and one may also include further layers of adjoints. When the underlying graph or 2-graph has only one object, the resulting categories can be viewed as monoidal categories with a partial trace. In this talk I will discuss various aspects of the structure of these categories.

This is joint work with Robert Dawson (Saint Mary's University) and Robert Paré (Dalhousie University).

References

[1] Robert Dawson, Robert Paré and Dorette Pronk, *Adjoining adjoints.* Adv. Math. **178**(2003), 99–140.

BRIAN REDMOND, University of Calgary, 2500 University Drive NW, Calgary, Alberta T2N 1N4
Polarized strong categories for lower complexity

I shall discuss the use of polarized strong categories as an abstract setting for investigating lower complexity computations. The aim of this work is to identify key structural properties for separating complexity classes. In particular, I shall highlight the role that distributivity plays in the separation of PTIME and PSPACE.

Joint work with Mike Burrell and Robin Cockett.

ROBERT ROSEBRUGH, Mount Allison University, Sackville, NB
Algebras and view updates

Database view updating can be seen as a lifting problem, so it is not surprising that fibrations arise. For $\mathbf{C}$ a category with products, and an object B, the sum functor $\mathbf{C}/B \longrightarrow \mathbf{C}$ is a left adjoint, and an algebra $(G\colon E \to B, P)$ for the generated monad on $\mathbf{C}/B$ has G essentially a projection (called a "lens" by Pierce).

When $\mathbf{C} = \mathbf{Cat}$ a lens $G\colon E \to B$ is an (op)fibration. On the other hand, taking the projection $(G, 1_B) \to B$ from the comma category is the functor part of a monad on $\mathbf{Cat}/B$. An algebra for $(-, 1_B)$ provides a good notion of a "partial lens". Furthermore, an opfibration has "universal translations". These provide a universal solution to the view updating problem when $G = W^*\colon \mathrm{Mod}(E) \to \mathrm{Mod}(V)$ for a view (sketch morphism) $W\colon V \to E$ in the Sketch Data Model.

We will also make remarks about how to interpret the lens notion in tensor categories.

PHIL SCOTT, Dept. of Math, U. Ottawa
Recent Results in Partially Traced Tensor Categories

An abstract theory of traces in monoidal categories was introduced by Joyal, Street, and Verity in 1996 (Proc. Camb. Phil. Soc. **119**, 447–468). Their notion covered a wide range of examples from algebra, knot theory, and algebraic topology to fixed point operators and models of feedback in theoretical computer science and logic. Since then, several groups of authors have investigated partially traced monoidal categories, in which the trace operator is only partially defined. These arise in many settings, from tensored $*$-categories to categorical logic and theoretical computer science. Recently, Esfan Haghverdi and I developed a new notion of partially traced tensor category arising from the proof theory of linear logic (Girard's Geometry of Interaction program), but which seems to have independent mathematical interest. I shall give a survey of many examples of these partial traces, including some recent work by Octavio Malherbe giving new classes of examples arising from monoidal subcategories of traced categories. If time permits, we will mention various recent connections of this work with Freyd's theory of paracategories.

ROBERT SEELY, McGill University and John Abbott College, Montreal
Faà di Bruno Categories

In several papers, Blute, Cockett and Seely have described a categorical approach to differential calculus based on intuitions (due to T. Ehrhard) from linear logic. Specifically, they presented a comonadic setting, whose functor is a differential operator, whose base maps are linear and whose coKleisli maps are smooth. There are two complementary approaches to differential categories: differential categories (where the emphasis is on the base category with a differential comonad) and Cartesian differential categories (where the emphasis is reversed, presenting the category of smooth maps inside which lives the category of linear maps). There are natural connections between these two notions, but they are not equivalent (unless considerably strengthened).

In the Cartesian differential categorical context, the structure of the chain rule gives rise to a fibration, the "bundle category". In this talk I shall generalise this to the higher order chain rule (originally developed in the traditional setting by Faà di Bruno in the nineteenth century); given any Cartesian differential category X, there is a "higher-order chain rule fibration" $\mathrm{Faa}(X) \to X$ over it. In fact, Faa is a comonad (over the category of Cartesian left semi-additive categories); the main theorem is that the coalgebras for this comonad are precisely the Cartesian differential categories.

Joint work with Robin Cockett.

MICHAEL SHULMAN, University of Chicago, 5734 S. University Ave, Chicago, IL 60637, USA
Extraordinary multicategories

The theory of 2-categories gives a convenient context in which to study analogues of natural transformations in different contexts. However, a satisfactory theory which does the same for "extraordinary" natural transformations, such as evaluation and coevaluation in a closed monoidal category, has proven elusive. One possibility is an "autonomous bicategory" or "autonomous double category," such as those consisting of categories and profunctors. However, this introduces additional structure which is unnecessary for the particular purpose. I will present a structure called an "extraordinary 2-multicategory," which is the "minimal extension" of a 2-category including extraordinary natural transformations. It is an example of a "generalized multicategory," and turns out to be related to autonomous double categories in much the same way that ordinary multicategories are related to monoidal categories.

MICHAEL WARREN, University of Ottawa, Department of Mathematics and Statistics, 585 King Edward Ave., Ottawa, Ontario K1N 6N5
Strict intervals in monoidal categories

By a *strict interval* I in a monoidal category $\mathcal{E}$ we mean a cocategory object such that the object of coobjects is the tensor unit. For example, the free category $\mathbf{2}$ on the graph with two distinct vertices and a single edge between them is (the object of coarrows of) a strict interval with respect to the cartesian monoidal structure on the category $\mathbf{Cat}$ of small categories, and it is well-known that the familiar 2-category structure on $\mathbf{Cat}$ is induced by this strict interval. In general, when $\mathcal{E}$ possesses a strict interval I and suitable additional structure, there is an induced 2-category structure on $\mathcal{E}$ and it is possible to say a good deal about this 2-category structure on the basis of simply examining the properties of I itself. For instance, we can characterize completely those I which induce on $\mathcal{E}$ a finitely bicomplete 2-category structure. In this talk we will describe these and related facts regarding the 2-categorical and homotopy theoretic properties of monoidal categories $\mathcal{E}$ which possess a strict interval.

RICHARD WOOD, Dalhousie University
Tensor Products of Sup Lattices

The category sup of complete lattices and sup-preserving functions is well known to underlie a tensor category for which the tensor product classifies functions of two variables that preserve suprema in each variable separately. It is classical, after the work of Joyal and Tierney, that sup as a tensor category is somewhat similar to the tensor category of abelian groups. In particular, although much older still, $L \otimes M$ is the quotient of the free sup-lattice $\mathcal{P}(|L| \times |M|)$ by the smallest congruence $\sim$ for which $(\bigvee_i l_i, m) \sim \bigvee_i (l_i, m)$ and $(l, \bigvee_i m_i) \sim \bigvee_i (l, m_i)$.

The arrow $\mathcal{P}(|L| \times |M|) \longrightarrow L \otimes M$ in sup has a fully faithful right adjoint so that it is natural to look for a description of $L \otimes M$ as a full reflective subobject of $\mathcal{P}(|L| \times |M|)$ in ord, the 2-category of ordered sets, order-preserving functions, and inequalities. In fact, it is even more natural to pursue such an approach if we replace the category set of sets and the adjunction $\mathcal{P} \dashv |-| \colon \mathrm{sup} \longrightarrow \mathrm{set}$ by the adjunction $\mathcal{D} \dashv |-| \colon \mathrm{sup} \longrightarrow \mathrm{ord}$. The talk explores this possibility and its extension to tensor products for algebras of KZ-doctrines more generally.

Joint work with Toby Kenney.