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Weakly closed structures on higher-dimensional categories

In this talk, I return to the point of view of Eilenberg's and Kelly's "Closed Categories", which considers the closed structure, based on hom-objects, as primary, and the tensor product as secondary. The motivating example is Gray, the 3-dimensional category that can be defined as a closed category whose objects are (small) 2-categories, and whose arrows are 2-functors, by specifying the hom-objects in a relatively simple way (involving pseudo-natural transformations), without mentioning the (in fact, available) symmetric monoidal structure. There are several other examples of categories, with objects certain kinds of higher-dimensional categories, which have a natural candidate for a concept of internal hom, and which turn out to be closed categories, in a weakened sense at least. One is Gray-Cat, whose objects are Gray-categories. In a paper from 1999, Sjoerd Crans defines a tensor product on Gray-Cat, of which he shows that it does not carry a closed structure. My approach is of the opposite kind to Crans's in starting with a (weakly) closed structure that is not only simple but also useful in developing 2-dimensional category theory, for instance 2-dimensional Gabriel–Ulmer duality. Another example has objects tricategories. This last example has been used to give a proof of a strong form of the coherence theorem for tricategories.