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Meditations on Arens Multiplication

Between the years 1945, when Eilenberg and MacLane officially made public the notions of category, functor, natural transformation, and natural equivalence, and 1965, when Eilenberg and Kelly released their definitive opus on closed and monoidal categories, there were other tentative steps towards a realization of the concept of closed category, steps focussing, for the most part, on Banach spaces or more general sorts of spaces of interest to Analysts: there were the 1960-vintage Doklady and Uspechi articles of Fuks, Shvarts, and Mityagin on duality of functors on Banach spaces; the 1955 work of Grothendieck on topological tensor products; Robert Schatten's 1950 Annals Study, *A Theory of Cross-Spaces*; and (the focus of this presentation) the 1950 work of Richard Arens (published in 1951) on what has come to be called the Arens multiplication on the second conjugate A^{**} of a Banach algebra A , a construction relying, conceptually, on what Arens there called *phyla*, entities which correspond, grosso modo, to a kind of blend of what are known today as (associative) closed monoidal categories and today's multilinear categories.

The aim of this presentation is to publicize more widely this 60-year-old work of Arens that so unexpectedly pre-figured today's closed monoidal categories, and to bring into relief some of the as yet unsolved problems it both posed then and suggests today.

References

- [1] Samuel Eilenberg and Saunders MacLane, *General Theory of Natural Equivalences*. Trans. Amer. Math. Soc. **58**(1945), 231–294.
- [2] Richard Arens, *Operations induced in function classes*. Monatsh. Math. **55**(1951), 1–19.