Spectral Methods in the Analysis of Differential Equations
Méthodes spectrales en analyse des équations différentielles
(Org: **Almut Burchard** and/et **Marina Chugunova** (Toronto))

PAUL BINDING, U. of Calgary
Some Sturm–Liouville type results for the p-Laplacian

A review will be given of some recent work on nonlinear (but homogeneous) generalisations of the Sturm–Liouville problem involving the p-Laplacian.

Many (but not all) of the usual questions are understood when the problem is regular, definite and with separated boundary conditions, but interesting issues arise when these assumptions are dropped, individually or in combination.

ALMUT BURCHARD, University of Toronto, Department of Mathematics, 40 St. George Street, Toronto, Ontario M5S 2E4
On computing the instability index of certain non-selfadjoint operators

This talk will discuss recent joint work with M. Chugunova on the problem of finding the instability index of certain non-selfadjoint fourth order differential operators. The work is motivated by linearizations of coating and rimming flows, where a thin fluid film moves on a horizontal rotating cylinder.

Our main result states that the instability index of such operators is determined by its restriction to a finite-dimensional space of trigonometric polynomials, and provides a condition on the dimension of this space. The proof uses Lyapunov's method to associate the differential operator with a quadratic form whose maximal positive subspace has dimension equal to the instability index. The quadratic form is determined by a solution of Lyapunov's equation, which here takes the form of a fourth order linear PDE in two variables. Elliptic estimates for the solution of this PDE play a key role.

MARINA CHUGUNOVA, University of Toronto
On stability of waves in liquid films on vibrating substrates

The effect of vibration on the stability of liquid films is of primary importance for many industrial applications. Using the model derived by Shklyaev, Alabuzhev, and Khenner, we show that all periodic and solitary-wave solutions of this equation are unstable regardless of their parameters. Some of the solitary waves, however, are metastable (i.e., still unstable, but with extremely small growth rates) and, thus, can persist without breaking up for a very long time. We study the dynamics of these waves analytically and numerically.

Joint work with E. Benilov (University of Limerick).

YUFANG HAO, Department of Applied Math, University of Waterloo, Waterloo, ON N2L 3G1
Spectra of Self-Adjoint Extensions of a Symmetric Operator with Deficiency Indices $(1,1)$ and Application in Sampling Theory

This talk will discuss the spectra of self-adjoint extensions of unbounded simple symmetric operators T with deficiency indices $(1,1)$. Any $(1,1)$-symmetric operator T has a $U(1)$-family of self-adjoint extensions, say $T(\alpha)$, for $0 \leq \alpha < 2\pi$. If one of self-adjoint extension, say $T(0)$ at $\alpha = 0$, has a set of discrete spectra with no accumulation point, then each self-adjoint operator $T(\alpha)$ has a set of discrete eigenvalues $\{t_n(\alpha)\}_{n=-\infty}^{\infty}$, and together they cover the real line exactly once. Further, given the spectra of $T(0)$ and the corresponding derivatives defined as $t_n'(\alpha) = \frac{dt_n(\alpha)}{d\alpha}$, one can obtain an explicit formula for computing the eigenvalues of all other self-adjoint extensions of T. This provides a computational realization of the abstract Cayley transform. As an application, we will show results on a new generalized sampling theory.

ILLIA KARABASH, University of Calgary, 2500 University Drive NW, Calgary, Alberta T2N 1N4
Extension of Molchanov's oscillation criterion to nonlinear ODE of p-Laplacian type

It is well known that Molchanov's criterion for discreteness of spectrum of a linear Sturm–Liouville operator $-d^2/dx^2 + q$ acting in $L^2(0,\infty)$ and having semi-bounded potential $q(x) \geq c$ can be reformulated in the terms of oscillation: the equation $-y''(x) + q(x)y(x) = \lambda y(x)$ is non-oscillatory for all $\lambda \in \mathbb{R}$ if and only if $\lim_{x\to\infty} \int_x^{x+\varepsilon} q = +\infty$ for any $\varepsilon > 0$.

It will be shown that this oscillation criterion holds for the equation

$$-\Big(\big(\operatorname{sgn} y'(x)\big)|y'(x)|^{p-1}\Big)' = (p-1)\big(\lambda - q(x)\big)\big(\operatorname{sgn} y(x)\big)|y(x)|^{p-1}, \quad x \in (0,\infty),$$

with $q(x) \geq c$ and $p > 1$, which is nonlinear for $p \neq 2$. The part 'if' was proved in Binding and Browne (2008). The main aim of the talk is the inverse implication 'only if'. It is also planned to discuss several recent related results.

The research is partially supported by the PIMS Post-Doctoral Fellowship at the University of Calgary. The talk is based on joint research with Paul Binding.

CHOI MAN-DUEN, University of Toronto, Toronto, Canada
Magic in non-commutative computation

Suddenly, the era of quantum computers arrives, where non-commutative matrix analysis will play a central role in many concrete applications.

Herein, I will show the hidden truth/myth in some sorts of magical computation, in connections to operator inequalities.

ANGELO MINGARELLI, Carleton

BORIS MITYAGIN, The Ohio State University
Convergence of spectral decompositions of Hill operators with trigonometric polynomial potentials

We consider the Hill operator

$$Ly = -y + v(x)y, \quad 0 \leq x \leq \pi,$$

subject to periodic or antiperiodic boundary conditions, with potentials v which are trigonometric polynomials with nonzero coefficients, of the form

$$\begin{array}{ll} (i) & a\,e^{-2ix} + b\,e^{2ix}; \\ (ii) & a\,e^{-2ix} + B\,e^{4ix}; \\ (iii) & a\,e^{-2ix} + A\,e^{-4ix} + b\,e^{2ix} + B\,e^{4ix}. \end{array}$$

Then the system of eigenfunctions and (at most finitely many) associated functions is complete but it is not a basis in $L^2([0,\pi],C)$ if $|a| \neq |b|$ in the case (i), if $|A| \neq |B|$ and neither $-b^2/4B$ nor $-a^2/4A$ is an integer square in the case (iii), and it is never a basis in the case (ii) subject to periodic boundary conditions.

This is a joint work with Plamen Djakov; see arxiv 0911.3218.

GIDEON SIMPSON, University of Toronto, Toronto, ON
Applications of Sinc Spectral Methods to Solitary Waves

Described by Sir Edmund Whittaker as "a function of royal blood in the family of entire functions, whose distinguished properties separate it from its bourgeois brethren," the Cardinal Whittaker Sinc function, sinc, can serve as the basis of a spectral method for solving differential equations whose solutions are expected to be smooth, providing near exponential convergence.

In this talk, we briefly review properties of the sinc discretization and then discuss two applications to solitary wave problems. For such problems, sinc has proven very effective for two reasons. First, it naturally incorporates the typical boundary conditions of solitary waves (vanishing at infinity). Second, it naturally captures the algebraic structure of the solitary wave equations, allowing us to compute the generalized kernel of the linearized equations. We also highlight questions at the interface of analysis and numerical analysis that arise in the application of this method.

HANS W. VOLKMER, University of Wisconsin–Milwaukee
The eigenvalue at infinity of a Sturm–Liouville problem

An indefinite Sturm–Liouville problem may have the eigenvalue infinity. We will present two results on the eigenspace corresponding to this eigenvalue. The first one is important in order to decide on the completeness of root vectors. The second one concerns the negativity index at infinity that appears in connection with the invariant subspace theorem in Pontrjagin space.