**Representation Theory of Algebras**
**Théorie des représentations des algèbres**
(Org: **Ibrahim Assem** (Sherbrooke), **Thomas Brüstle** (Sherbrooke; Bishop's) and/et **Shiping Liu** (Sherbrooke))

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**IBRAHIM ASSEM**, Université de Sherbrooke
*Cluster Automorphisms*

We define a notion of cluster automorphism of a cluster algebra. We show that, in the acyclic case, the computation of the group of cluster automorphisms reduces to the computation of the automorphism group of the transjective component of the Auslander–Reiten quiver of the cluster category. As a result, we obtain these groups for the Dynkin and the euclidean types. We then consider the case of the cluster algebras arising from surfaces and relate the group of cluster automorphisms with the mapping class group of the surface.

This is a report on a joint work with Ralf Schiffler and Vasilisa Shramchenko.

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**FRAUKE BLEHER**, University of Iowa, Department of Mathematics, 14 MLH, Iowa City, IA 52242, USA
*Parameters for dihedral blocks with two simple modules*

Let $k$ be an algebraically closed field of characteristic $2$, let $G$ be a finite group with dihedral Sylow $2$-subgroups, and let $B$ be the principal block of $kG$. Assume that there are precisely two isomorphism classes of simple $B$-modules. The description by Erdmann of the quiver and relations of the basic algebra of $B$ is usually only given up to a certain parameter $c$ which is either $0$ or $1$. In this talk, we will show that $c = 0$ if there exists a central extension $\hat{G}$ of $G$ by a group of order $2$ such that the Sylow $2$-subgroups of $\hat{G}$ are generalized quaternion. As a special case, we obtain that $c = 0$ if $G = \mathrm{PGL}_2(\mathbb{F}_q)$ for some odd prime power $q$.

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**THOMAS BRÜSTLE**, Sherbrooke University
*Cluster Categories from Surfaces*

We consider in this talk the cluster category of a marked surface, explicitly describing the objects and the Auslander–Reiten structure in geometric terms. We further show that the objects without self-extensions correspond to curves without self-intersections.

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**JUAN CARLOS BUSTAMANTE**, Sherbrooke
*Special biserial algebras with no outer derivations*

Let $k$ be a field. Given a special biserial $A = kQ/I$, we investigate the relationships between its simple connectedness, the vanishing of its Hochschild cohomology groups, and some combinatorial data for $(Q, I)$.

A bound quiver $(Q, I)$ is said to be special biserial if it satisfies the following:

(i) every vertex in $Q$ is the source of at most two arrows and the target of at most two arrows;
(ii) given an arrow $\alpha\colon x \to y$ in $Q$, there is at most one arrow $\beta$ starting from $y$ such that $\alpha\beta \notin I$ and there is at most one arrow $\gamma$ arriving at $x$ such that $\gamma\alpha \notin I$.

Consider the special biserial algebra $A = kQ/I$, and let $S$ be the two-sided ideal of $A$ generated by classes of paths appearing in binomial relations, so that $A/S$ is a string algebra.

Moreover, denote by $\mathrm{HH}^i(A) := \mathrm{Ext}^i_{A-A}(A, A)$ the Hochschild cohomology groups of $A$ with coefficients in the bimodule ${}_AA_A$. The sum $\mathrm{HH}^*(A) := \bigoplus_{i\geqslant 0} \mathrm{HH}^i(A)$ is then a graded commutative ring for the Yoneda product and a graded Lie algebra for a bracket defined using composition of cochains.

An algebra $A$ is said to be simply connected if it has no proper Galois cover. This amounts to say that if $A \simeq kQ/I$ then the fundamental group $\pi_1(Q, I)$ is trivial.

The main result of this work is:

**Theorem** *Let $A = kQ/I$ be a special biserial algebra. Then the following conditions are equivalent.*

*(1) $A$ is simply connected of finite representation type;*
*(2) $\mathrm{HH}^1(A) = 0$;*
*(3) $\mathrm{HH}^*(A) = k$;*
*(4) $\dim_k S = \chi(Q)$.*

We shall discuss about the proof and some consequences.

This is a joint work with Ibrahim Assem, and Patrick Le Meur.

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**GRÉGOIRE DUPONT**, Université de Sherbrooke, Sherbrooke, QC J1K 2R1
*Geometric bases in cluster algebras*

In this talk I will explain how to construct linear bases in acyclic cluster algebras by means of geometric methods in representation theory of quivers.

I will introduce the notion of generic variables in an arbitrary acyclic cluster algebra and show that it provides an explicit basis in the affine case.

I will also provide a geometric realization of another kind of bases, putting into context the results of Sherman–Zelevinsky and Cerulli on "canonically positive bases".

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**ED GREEN**, Virginia Tech
*$2-d$ Koszul algebras*

We present some conditions relating when a quotient of a path algebra by a length homogeneous ideal generated in degree $d$ and its associated monomial algebra are $d$-Koszul.

We also study conditions on algebras that are quotients of path algebras where the ideal of relations can be generated by homogeneous elements of degrees $2$ and $d$. We investigate conditions that imply minimal graded projective resolutions of simple modules are 'nice'. We also study conditions that imply the $\mathrm{Ext}$-algebra of such an algebra is finitely generated.

This talk describes joint work with Eduardo Marcos.

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**BIRGE HUISGEN-ZIMMERMANN**, University of California at Santa Barbara
*Strongly tilting truncated path algebras*

We call a path algebra $\Lambda = KQ/I$ of a quiver $Q$ with coefficients in a field $K$ *truncated* if $I$ consists of all paths in $Q$ of a fixed length $L \geq 2$. Note that this class of algebras includes the finite dimensional split hereditary algebras, while, on the other hand, every finite dimensional split algebra occurs as a quotient of a truncated path algebra.

It is shown that for any truncated path algebra $\Lambda$, the subcategory $\mathcal{P}^{<\infty}(\Lambda-\text{mod})$ consisting of the objects of finite projective dimension in $\Lambda-\text{mod}$ is contravariantly finite in $\Lambda-\text{mod}$. Hence, due to Auslander and Reiten, there exists a (unique basic) tilting module $T$ which is Ext-injective in $\mathcal{P}^{<\infty}(\Lambda-\text{mod})$. If $\widetilde{\Lambda} = \mathrm{End}_\Lambda(T)^{\mathrm{op}}$ is the corresponding "strongly tilted algebra", its category $\mathcal{P}^{<\infty}(\text{mod}-\widetilde{\Lambda})$ is in turn contravariantly finite in $\text{mod}-\widetilde{\Lambda}$. The result is replicated as one moves on to the strongly tilted algebra of $\widetilde{\Lambda}$. We discuss the particularly strong connection between the structure of the representations of $\Lambda$ and those of its successive strongly tilted algebras.

The first part of the talk addresses joint research with Alex Dugas.

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**KIYOSHI IGUSA**, Brandeis University, Waltham, MA
*Spaced-out $m$-clusters*

"Spaced-out" clusters of type $A_n$ are cluster tilting objects in which no two objects are connected by an irreducible map. For those clusters, "internal mutations" are given by the octagon axiom for a triangulated category. I will describe the spaced-out clusters in the $m$-cluster category to type $A_n$ and show how the internal mutations are given by a generalization of the octagon axiom.

Joint work with Gordana Todorov.

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**MARK KLEINER**, Syracuse University, Department of Mathematics, Syracuse, NY 13244-1150
*Homological methods in the representation theory of partially ordered sets*

One of the methods in the theory of modules over finite dimensional algebras consists in reducing, in some sense, a problem about modules to a problem of linear algebra. Although such a reduction is not always fruitful, the method has had many applications, which have identified several important classes of problems of linear algebra. One of these classes, representations of partially ordered sets, admits certain combinatorial procedures that replace the given partially ordered set with a new one, having almost the same representations. Although the procedures, called differentiation algorithms, are crucial for applications to module theory, proving that the algorithms work involves rather cumbersome computations that treat the algorithms as ad hoc constructions and give no indication of their relationship to conceptual mathematics. The purpose of this talk is to interpret differentiation algorithms using classical methods of homological algebra.

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**PATRICK LE MEUR**, CMLA, ENS Cachan, CNRS, UniverSud, 61 Avenue du President Wilson, F-94230 Cachan, France
*Crossed-products of Calabi–Yau algebras by finite groups*

Calabi–Yau algebras were defined by Ginzburg as non-commutative analogues of coordinate rings of Calabi–Yau manifolds. In the representation theory of finite dimensional algebras, the Calabi–Yau (differential graded) algebras of dimension $3$ are key ingredients to construct $2$-Calabi–Yau categories (following the work of Amiot) which serve to categorify the cluster algebras defined by Fomin and Zelevinsky.

For example, $\mathbb{C}[X,Y,Z]$ is a Calabi–Yau algebra. So are the Weyl algebras, as proved by Berger.

When a finite group $G$ acts on an algebra $A$, the crossed-product algebra $A \rtimes G$ is often considered as a nice (smooth) replacement of the algebra of invariants $A^G$, the latter being more difficult to handle. For example, if $G$ is a finite subgroup of $\mathrm{SL}_3(\mathbb{C})$, then it acts on $\mathbb{C}[X,Y,Z]$. In this setting, Ginzburg proved that $\mathbb{C}[X,Y,Z] \rtimes G$ is Calabi–Yau.

In this talk we shall see to what extent this result still holds for the action of a finite group on a Calabi–Yau algebra. Some consequences in representation theory of finite dimensional algebras will be presented.

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**FRANTISEK MARKO**, The Pennsylvania State University, 76 University Drive, Hazleton, PA 18202, USA
*Algebra of supersymmetric polynomials and invariants of general linear supergroup*

In the case of characteristic zero, generators of the algebra of supersymmetric polynomials in the characteristic zero case were described by Stembridge. That result is related to a work of Kantor and Trishin who described generators of the algebra of invariants of general linear supergroup. In a joint work with A. N. Grishkov and A. N. Zubkov we described the generators of the algebra of supersymmetric polynomials and the algebra of invariants of general linear supergroup in the case of positive characteristic. I will give an overview of these results.

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**CHARLES PAQUETTE**, Université de Sherbrooke, 2500, boul. Université, Sherbrooke, QC J1K 2R1
*Auslander–Reiten theory for the representations over an infinite quiver*

Let $Q$ be an infinite quiver which is locally finite and such that the number of paths between two given vertices is finite. We study the Auslander–Reiten theory of the category $\mathrm{rep}(Q)$ of the locally finite dimensional representations over a field $k$. Let $\mathrm{rep}^+(Q)$ be the representations of $\mathrm{rep}(Q)$ which are finitely presented. With the additional condition that $Q$ has no left infinite paths, we describe the components of the Auslander–Reiten quiver of $\mathrm{rep}^+(Q)$. We find that all regular components are of shape $\mathbb{Z}\mathbb{A}_\infty$ or $\mathbb{N}^-\mathbb{A}_\infty$. The number of these components is finite if and only if $Q$ is of infinite Dynkin type. We also give a condition on $Q$ for when all regular components are of shape $\mathbb{N}^-\mathbb{A}_\infty$.

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**YVAN SAINT-AUBIN**, Yvan Saint-Aubin, Université de Montréal
*The Temperley–Lieb algebra, indecomposable representations and statistical physics*

Algebra has played a crucial role in modern physics, particularly through representation of groups and Lie algebras. Recently representations of associative algebras have also played an important role. I shall describe how the representation theory of the Temperley–Lieb algebra, an associative algebra, enters the description of phase transition in two-dimensional statistical physics.

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**RALF SCHIFFLER**, University of Connecticut
*Cluster algebras from surfaces*

Cluster algebras from surfaces are a special type of cluster algebras where the clusters of the cluster algebra are in bijection with triangulations of a Riemann surface with boundary and marked points. This class of cluster algebras contains for example the Dynkin types $A, D$ and the Euclidean types $\tilde{A}, \tilde{D}$, but these are only the easiest examples and the whole class is significantly bigger.

In this talk I will explain how one can use the surface to perform explicit computations in the cluster algebra, including explicit formulas for the cluster variables and positivity results.

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**GORDANA TODOROV**, Northeastern University, Boston, MA 02115, USA
*Functor categories, module categories and words in Coxeter groups*

As a natural extension of various works on $c$-sortable words and their relation to the functors on the derived category and modules over preprojective algebras, we consider also general words and their associated categories of functors and modules.

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**SONIA TREPODE**, Universidad Nacional de Mar del Plata
*Cluster tilted algebras with cyclically oriented quiver*

In this talk we study cluster-tilted algebras whose quiver is cyclically oriented. In this case an explicit description of the defining relations is given. For this kind of algebras, it is also shown that there exists an admissible cut and moreover that each arrow of the quiver is contained in an admissible cut. Furthermore, we show that if the endomorphism ring of an algebra of global dimension two over its cluster category, in the sense of Amiot, is cluster-tilted and has a cyclically oriented quiver, then the original algebra is a quotient by an admissible cut. In the case of cluster tilted algebras of Dynkin or extended Dynkin type, the connection is stronger and also the converse statement holds. Even more, in that case the original algebra is derived equivalent to the hereditary algebra.

Joint work with Michael Barot.

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**HELENE TYLER**, Department of Mathematics and Computer Science, Manhattan College
*A Comparison of Orderings for Preprojective Modules*

In 2004, C. M. Ringel published his first of several papers on the Gabriel–Roiter measure. This measure assigns to each module a rational number that depends on its submodule lattice. Like the Auslander–Reiten theory, the Gabriel–Roiter measure partitions the module category into three classes. This talk is a report on joint work with Markus Schmidmeier, in which we investigate the connections between the two partitions. In particular, we discuss the relationship among the orderings on the preprojective modules given by the Gabriel–Roiter measure, the shortest annihilating sequence of reflections, and the slope of a module.