Stability in Nonlinear Partial Differential Equations
Stabilité pour les équations aux dérivées partielles nonlinéaires
(Org: **Stephen Gustafson** (UBC) and/et **Dmitry Pelinovsky** (McMaster))

BERNARDO GALVAO-SOUSA, McMaster University, 1280 Main St. W, Hamilton, ON, L8S 4L8
Thin Film Limits for Ginzburg–Landau with Strong Applied Magnetic Fields

We study thin-film limits of the full three-dimensional Ginzburg–Landau model for a superconductor in an applied magnetic field oriented obliquely to the film surface. We obtain Γ-convergence results in several regimes, determined by the asymptotic ratio between the magnitude of the parallel applied magnetic field and the thickness of the film. Depending on the regime, we show that there may be a decrease in the density of Cooper pairs. We also show that in the case of variable thickness of the film, its geometry will affect the effective applied magnetic field, thus influencing the position of vortices.

STEPHEN GUSTAFSON, University of British Columbia, 1984 Mathematics Rd., Vancouver, BC V6T 1Z2
Singularities and asymptotics for some geometric nonlinear Schroedinger equations

I will describe results on singularity (non-)formation and stability, in the energy-critical 2D setting, for some nonlinear Schroedinger-type systems of geometric origin—the Schroedinger map and Landau–Lifshitz equation—which model dynamics of ferromagnets and liquid crystals.

SLIM IBRAHIM, University of Victoria
Strichartz type estimates and application to a 2D energy critical NLW in a bounded domain

In this work, we establish an appropriate 2D Strichartz type estimate for the linear wave equation set on a bounded domain with either Dirichlet or Neumann type boundary conditions. The proof follows Burq–Lebeau–Planchon work in 3D and solely based on spectral projection estimates due to Sogge. Our Strichart estimate enables us to solve the nonlinear problem with exponential nonlinearity. We define a trichotomy for the cauchy problem, prove the wellposedness on two sides of the trichotomy and a sort of instability on the the last side.

This is a joint work with R. Jrad.

KAY KIRKPATRICK, Courant Institute, NYU, 251 Mercer St., New York, NY 10012
Bose–Einstein condensation: from many quantum particles to a quantum "super-particle" and beyond

Near absolute zero, a gas of quantum particles can condense into an unusual state of matter, called Bose–Einstein condensation, that behaves like a giant quantum particle. It's only recently that we've been able to make the rigorous connection between the physics of the microscopic dynamics and the mathematics of the macroscopic model, the cubic nonlinear Schrodinger equation (NLS). I'll discuss joint work with Benjamin Schlein and Gigliola Staffilani on two-dimensional cases of Bose–Einstein condensation–and the periodic case is especially interesting, because of techniques from analytic number theory and applications to quantum computing. As time permits, I'll also mention work in progress on computational quantum many-body systems and phase transitions for the invariant measures of the NLS.

EDUARD KIRR, University of Illinois at Urbana-Champaign
Stability and bifurcations of large bound states in nonlinear Schroedinger equation

I will present recent necessary and sufficient conditions for the existence of bifurcation points along ground state and excited state branches in nonlinear Schroedinger equations. The possible types of bifurcations and their effect on the dynamical stability of the bound states will also be discussed.

This is joint work with D. Pelinovsky (McMaster), P. Kevrekidis (U. Mass.) and V. Natarajan (U. of Illinois).

EVA KOO, University of British Columbia, Vancouver, BC
Asymptotic stability of small solitary waves for nonlinear Schrödinger equations with electromagnetic potential in $\mathbb{R}^3$

Consider the nonlinear magnetic Schrödinger equation for $u\colon \mathbb{R}^3 \times \mathbb{R} \to \mathbb{C}$,

$$iu_t = (i\nabla + A)^2 u + Vu + g(u), \quad u(x,0) = u_0(x),$$

where $A\colon \mathbb{R}^3 \to \mathbb{R}^3$ is the magnetic potential, $V\colon \mathbb{R}^3 \to \mathbb{R}$ is the electric potential, and $g = \pm|u|^2 u$ is the nonlinear term. We will show that under suitable assumptions on A and V, if the initial data u_0 is small enough in H^1, then the solution $u(x,t)$ decomposes uniquely into a standing wave part and a dispersive part which scatters.

QIUPING LU, Centro de Modelamiento Matematico, U. de Chile
Compactly supported solutions of a class of semilinear elliptic equations

A class of semilinear elliptic equations, which describes time-independent solutions to a degenerate parabolic equation modeling population dynamics, is studied. Under suitable assumptions all solutions are compactly supported, moreover, multiplicity of solutions is shown by the methods of variations.

DMITRY PELINOVSKY, McMaster University
Excited states in the Thomas–Fermi limit

Excited states of Bose–Einstein condensates are considered in the Thomas–Fermi limit, in the framework of the Gross–Pitaevskii equation with repulsive inter-atomic interactions in a harmonic potential. The relative dynamics of dark solitons (density dips on the localized condensates) with respect to the harmonic potential and to each other is approximated using the averaged Lagrangian method and the Lyapunov–Schmidt reductions. This permits a complete characterization of the equilibrium positions of the dark solitons as a function of the chemical potential parameters. It also yields an analytical handle on the oscillation frequencies of dark solitons around such equilibria. The asymptotic predictions are generalized for an arbitrary number of dark solitons and are corroborated by numerical computations for two- and three-soliton configurations.

PATRICK REYNOLDS, Queen's University, Kingston, Ontario
Criteria for certain systems of PDEs to be Hamiltonian

A system of hydrodynamic type is a system of quasilinear first-order PDEs; the quasilinear nature, remarkably and beautifully, allows us to study such systems using finite-dimensional differential-geometric methods. To say such a system is Hamiltonian is to say that it is composed of some Poisson bracket and some Hamiltonian function. The motivating question is: given a system of hydrodynamic type, how can we determine whether or not it is Hamiltonian? We certainly can't test all Poisson brackets and all Hamiltonian functions! I'll present a recent answer to this question for systems of hydrodynamic type with three equations.

ANTON SAKOVICH, McMaster University, Department of Mathematics, Hamilton, ON, L8S 4K1
Internal modes of discrete solitons near the anti-continuum limit of the dNLS equation

Discrete solitons of the discrete nonlinear Schrödinger (dNLS) equation become compactly supported in the anti-continuum limit of the zero coupling between lattice sites. Eigenvalues of the linearization of the dNLS equation at the discrete soliton determine its spectral and linearized stability. All unstable eigenvalues of the discrete solitons near the anti-continuum limit were characterized earlier for this model. Here we analyze the resolvent operator and prove that it is uniformly bounded in the neighborhood of the continuous spectrum if the discrete soliton is simply connected in the anti-continuum limit. This result rules out existence of internal modes (neutrally stable eigenvalues of the discrete spectrum) of such discrete solitons near the anti-continuum limit.

GIDEON SIMPSON, University of Toronto, Toronto, ON

Spectral Analysis of Matrix Hamiltonian Operators

We study the spectral properties of matrix Hamiltonians generated by linearizing nonlinear Schrödinger equations about soliton solutions. Using a hybrid analytic-numerical proof, we show that there are no embedded eigenvalues for the 3-dimensional cubic nonlinearity, and other nonlinearities. Though we focus on the 3d cubic problem, the goal of this work is to present a new, robust, algorithm for verifying the spectral properties needed for stability analysis. We also present several cases for which our approach is inconclusive and speculate on ways to extend the method.

This is joint work with J. L. Marzuola (Columbia University).

HOLGER TEISMANN, Acadia University

Local controllability of a Bose–Einstein condensate in a time-varying box

In this talk we consider the "condensate-in-time-varying-box" problem in one space dimension,

$$i\mathbf{u}_t = -\mathbf{u}_{xx} - |\mathbf{u}|^2\mathbf{u} \tag{1a}$$

$$\mathbf{u}(t,0) = \mathbf{u}\big(t, L(t)\big) = 0. \tag{1b}$$

Taking the length, $L(t)$, of the box to be the control, we show that (1a), (1b) is controllable in the vicinity of the nonlinear ground state.

This is joint work with Karine Beauchard (Cachan) and Horst Lange (Cologne).

FRIDOLIN TING, Lakehead University, 955 Oliver Road, Thunder Bay, Ontario P7B 5E1

Dynamic stability of multi-vortex solutions to Ginzburg–Landau equations with external potential

We consider the dynamic stability of pinned multi-vortex solutions to Ginzburg–Landau equations with external potential in $\mathbb{R}^2$. For a sufficiently small external potential with widely spaced non-degenerate critical points, there exists a perturbed multi-vortex (pinned) solution whose vortex centers are near critical points of the potential. We show that multi-vortex solutions which are concentrated near local maxima of the potential are orbitally stable w.r.t. gradient and Hamiltonian dynamics.

VITALI VOUGALTER, University of Toronto, Department of Mathematics, Toronto, ON M5S 2E4

On the solvability conditions for the diffusion equation with convection terms

Linear second order elliptic equation describing heat or mass diffusion and convection on a given velocity field is considered in $\mathbb{R}^3$. The corresponding operator L may not satisfy the Fredholm property. In this case, solvability conditions for the equation $Lu = f$ are not known. In this work, we derive solvability conditions in $H^2(\mathbb{R}^3)$ for the non self-adjoint problem by relating it to a self-adjoint Schrödinger type operator, for which solvability conditions are obtained in our previous work.