
Inverse Problems in Partial Differential Equations
Problèmes inverses pour les équations aux dérivées partielles
(Org: **Adrian Nachman** (Toronto))

GUILLAUME BAL, Columbia University, New York, NY

Inverse Transport Problems and Photoacoustics

Inverse transport consists of reconstructing the optical parameters in a transport equation from knowledge of a measurement operator. We review several uniqueness and stability results obtained in the context of various boundary measurements as they arise, e.g., in optical tomography, a medical imaging modality. Accurate numerical reconstructions obtained by carefully capturing the singularities of the measurement operator are also briefly presented. I will also present recent results obtained for the inverse transport problem with internal controls as they arise in the application of photoacoustic tomography, a recent hybrid medical imaging modality that combines the large contrast observed in optical parameters with the high resolution of ultrasounds.

These are joint works with Alexandre Jollivet, Francois Monard, and Gunther Uhlmann.

ALLAN GREENLEAF, University of Rochester, Rochester, NY 14618, USA

Linearized seismic inversion in the presence of caustics

Linearized high frequency seismic inversion attempts to determine the singularities of the sound speed in the subsurface of the Earth from singularities of pressure field data recorded at the surface. If the ray geometry for the smooth background sound speed has no caustics, then Beylkin showed that the forward map is a “classical” Fourier integral operator, associated with a canonical graph, and the resulting normal operator is a pseudodifferential operator. Nolan and Symes began the examination of the consequences of caustics for the operator theory. I will discuss what happens both for folds and the next most commonly encountered type of caustics, namely cusps. The work described is due to various subsets of Raluca Felea, Malabika Pramanik and myself.

ELDAD HABER, UBC

A computational method for the Monge Kantorovich Problem

In this talk we discuss the fluid mechanics interpretation of the optimal mass transport problem. This formulation was proposed by Benamou and Brenier but was considered to be too expensive computationally to be applied for large scale problems. In this talk we show that by using consistent discretization techniques, advanced optimization methods and a multigrid solver we are able to reduce the computational cost of the problem such that it can be efficiently solved using modest computational resources.

MICHAEL LAMOUREUX, University of Calgary, Dept. Math. & Stat.

Gabor multipliers in imaging

Gabor multipliers are a class of linear operators representing localized Fourier multiplier which operate on signals in the time-frequency domain. We have developed this family of operators for rapid numerical approximation to solutions of PDEs, for use in the solution of the inverse problems that arise in seismic imaging. Analogous to the pseudodifferential calculus, we demonstrate an approximate functional calculus for Gabor multipliers on generalized frames and indicate their application to source-signature separation in Q-attenuated seismic signal propagation, deconvolution, and seismic migration.

This is joint work with Drs. Peter Gibson (York) and Gary Margrave (Calgary).

SHARI MOSKOW, Drexel University, Philadelphia, PA

Convergence of the inverse Born series for the Calderon problem

We analyze the inverse Born series for solving the inverse conductivity problem. In previous work the inverse series was used to develop fast image reconstruction algorithms in optical tomography. Here we study its use in EIT and characterize the convergence and stability in this context. We demonstrate its effectiveness with numerical reconstructions.

This is joint work with S. Arridge and J. Schotland.

PETRI OLA, University of Helsinki, Helsinki, Finland

Impedance tomography with an imperfectly known boundary

Electrical impedance tomography (EIT) aims to reconstruct the electric conductivity inside a physical body from current-to-voltage measurements at the boundary of the body. In practical EIT one often lacks exact knowledge of the domain boundary, and inaccurate modeling of the boundary causes artifacts in the reconstructions. A novel method to overcome this difficulty is discussed. The first step is to determine the minimally anisotropic conductivity in a model domain reproducing the measured EIT data. This is based on a classical result due to K. Strebel on the existence of extremal quasiconformal mappings. The algorithm is applied to simulated noisy data from a realistic electrode model. Also, we present few observations on this question in three dimensions.

MIKKO SALO, University of Helsinki

The Calderón problem on Riemannian manifolds

We consider the imaging of anisotropic materials by electrical measurements. This inverse problem arises in Electrical Impedance Tomography (EIT), which has been proposed as a diagnostic method in medical imaging and nondestructive testing. The mathematical model is the anisotropic Calderón problem, which consists in determining a matrix of coefficients in an elliptic equation from boundary measurements of solutions.

In geometric terms, the problem is to determine a Riemannian metric from Cauchy data of harmonic functions on a manifold. Our approach is based on Carleman estimates. We characterize those Riemannian manifolds which admit a special limiting Carleman weight. By using these weights we construct complex geometrical optics solutions to elliptic equations, and prove uniqueness results in inverse problems for a class of Riemannian manifolds.

This is joint work with D. Dos Santos Ferreira (Paris 13), C. Kenig (Chicago), and G. Uhlmann (Washington).

BENJAMIN STEPHENS, Washington

ALEX TAMASAN, University of Central Florida

Local reconstruction of electrical conductivity from incomplete interior data

This talk concerns the problem of conductivity reconstruction from partial interior data of the magnitude of the current density field. The method is based on the tracing of geodesics that join pairs of equipotential boundary points.

This is joint work with Adrian Nachman and Alexander Timonov.

LEO TZOU, Stanford University, Dept. Math.

Inverse Problems for the Schrödinger Operator on Riemann Surfaces

We show that on a smooth compact Riemann surface with boundary (M_0, g) the Dirichlet-to-Neumann map of the Schrödinger operator $\Delta_g + V$ determines uniquely the potential V . This seemingly analytical problem turns out to have connections with

ideas in symplectic geometry and differential topology. We will discuss how these geometric features arise and the techniques we use to treat them.

We will also discuss the problem of inverse scattering for the Schrödinger operator on noncompact surfaces with Euclidean ends. Here again the topology of the manifold plays a significant role. These topological obstructions suggest that counter-examples to identifiability exist on surfaces of many genus.

This is joint work with Colin Guillarmou of CNRS Nice. The speaker is partially supported by NSF Grant No. DMS-0807502 during this work.