Group Actions and Their Invariants
Actions de groupes et leurs invariants
(Org: **H E A Eddy Campbell** (UNB), **Jianjun Chuai** (MUN) and/et **David Wehlau** (RMC; Queen's))

HARM DERKSEN, University of Michigan
Approximate Categories for the Graph Isomorphism Problem

An isomorphism problem such as the graph isomorphism problem can be formulated as an orbit problem: Given a linear algebraic group G over a field k, a representation V, and two elements $v, w \in V$, do v, w lie in the same G-orbit? For every d we will construct a k-category $C_d(V)$ such that elements in V are objects of $C_d(V)$. If two elements $v, w \in V$ have the same G-orbit, then v and w are isomorphic objects in $C_d(V)$. There exists an efficient algorithm to test whether two objects in $C_d(V)$ are isomorphic. Applied to graphs, this yields a polynomial time algorithm which is often able to distinguish non-isomorphic graphs. The algorithm is at least as good as the higher-dimensional Weisfeiler–Lehman algorithm. Cai, Fürer and Immerman constructed graphs that cannot be distinguished in polynomial time with the Weisfeiler–Lehman method. For $k = \mathbf{F}_2$, our algorithm can distinguish these kind of graphs.

EMILIE DUFRESNE, MATCH, Universität Heidelberg, Germany
Additive group actions in positive characteristic

Roberts, Freudenburg, and Daigle and Freudenburg have given the smallest counterexamples to Hilbert's fourteenth problem. Each arises as the ring of invariants of an additive group action on a polynomial ring over a field of characteristic zero, and thus, each corresponds to the kernel of a locally nilpotent derivation. In positive characteristic, additive group actions correspond to locally finite iterative higher derivations, a more restrictive notion. We set up characteristic-free analogs of the three examples mentioned above, and show that, contrary to characteristic zero, in every positive characteristic, the invariant rings are finitely generated.

Joint work with Andreas Maurischat.

JONATHAN ELMER, Queen Mary, University of London
On the depth of separating algebras of finite groups

Suppose G is a finite group acting linearly on a k-vector space V and consider the ring of invariants $k[V]^G$. A separating algebra is a subalgebra A of $k[V]^G$ with the same orbit separation properties.

There has been much recent interest in separating algebras, as it is often easier to compute generating sets for separating algebras rather than their parent invariant rings. In the non-modular case, $k[V]^G$ is a Cohen–Macaulay ring, and so the inequality $\operatorname{depth}(A) \leq \operatorname{depth}(k[V]^G)$ holds trivially for all separating algebras $A \subset k[V]^G$. In this talk we will give examples in the modular case where this inequality also holds.

GENE FREUDENBERG, Western Michigan University, Kalamazoo, MI 49008, USA
Locally nilpotent derivations of rings with roots adjoined

We study $\mathbb{G}_a$-actions on certain affine varieties V which admit a dominant morphism to affine space $\mathbb{A}^n$. In particular, V is a variety whose coordinate ring B is the simple algebraic extension of the polynomial ring R defined by $B = R[z]$, where $z^n \in R$ for $n \geq 2$. In many cases, our methods show either that V admits no non-trivial $\mathbb{G}_a$-actions, or that the only $\mathbb{Z}_n$-homogeneous actions are those which lift from $\mathbb{A}^n$. Important examples include the Russell cubic threefold $X \subset \mathbb{A}^4$, defined by $x + x^2y + z^2 + t^3 = 0$, and the Pham–Brieskorn varieties $V \subset \mathbb{A}^n$, defined by $x_1^{e_1} + \cdots + x_n^{e_n} = 0$ for integers $e_i \geq 2$. By studying the locally nilpotent derivations (LNDs) of $B = R[z]$ relative to the affine ring R, we reveal a surprisingly effective general approach to understanding large classes of these varieties. In particular, our work features:

(1) criteria to determine that certain rings are rigid;
(2) a concise new proof that the Russell cubic X is not isomorphic to affine space;
(3) proofs for rigidity for a large family of Pham–Brieskorn threefolds and related varieties; and
(4) numerical criteria to determine when a homogeneous LND of the polynomial ring $A[x_1, \ldots, x_n]$ over A contains a variable x_i in its kernel.

A number of important open problems will also be discussed.

This talk will discuss joint work with Lucy Moser-Jauslin.

FRANK GROSSHANS, West Chester University, West Chester, PA 19383
The resolvent sextic of McClintock

Since Lagrange's seminal work in 1770, resolvent sextics have been used to study the quintic equation. One of the most interesting is the "noteworthy covariant resolvent discovered by Perrin and independently by McClintock" [L. E. Dickson]. We shall discuss the remarkable properties of this resolvent in light of the invariant theory of F_{20}, the Frobenius group of order 20 acting as a group of permutations on five variables, and SL_2 acting on binary forms.

LOEK HELMINCK, North Carolina State

HARLAN KADISH, University of Michigan, Mathematics Department, 2074 East Hall, 530 Church Street, Ann Arbor, MI 48109, USA
Polynomial Bounds for Invariant Functions Separating Orbits

Consider the representations of an algebraic group G. In general, polynomial invariant functions may fail to separate orbits. The invariant subring may not be finitely generated, or the number and complexity of the generators may grow rapidly with the size of the representation. We instead study "constructible" functions defined by straight line programs in the polynomial ring, with a new "quasi-inverse" that computes the inverse of a function where defined.

We write straight line programs defining constructible functions that separate the orbits of G. The number of these programs and their length have polynomial bounds in the parameters of the representation.

GREGOR KEMPER, Universität München
Invariants of a Vector and a Covector

For an action of a group G on an $\mathbf{F}$-vector space V, we consider the invariant ring $\mathbf{F}[V \oplus V^*]^G$. We are particularly interested in the case where $V = \mathbf{F}_q^n$ and G is the group U_n of all upper unipotent matrices or the group B_n of all invertible upper triangular matrices.

In fact, we determine $\mathbf{F}[V \oplus V^*]^G$ for $G = U_n$ and $G = B_n$. The result is a complete intersection for all values of n and q. We get explicit lists of generating invariants and the relations between them. This makes an addition to the rather short list of "doubly parametrized" series of group actions whose invariant rings are known to have a uniform description.

This talk is about joint work with Cédric Bonnafé.

MARTIN KOHLS, Technische Universität München Zentrum Mathematik, M11 Boltzmannstr. 3, D-85748 Garching, Germany
On degree bounds for separating invariants

Degree bounds for invariants are an old topic. The Noether bound says that in the non-modular case, invariants up to degree at most the group order generate the invariant ring. This bound has been sharpened in many ways. Today, many people prefer to consider separating invariants, instead of a full set of generators. This has many advantages. For example, the Noether bound always holds for separating invariants, even in the modular case. The goal of this talk is to give similar sharpenings for the Noether bound for separating invariants, as they exist for generating invariants. One result is that for p-groups in characteristic p, the Noether bound for separating invariants is sharp.

UGUR MADRAN, Izmir University of Economics, Izmir, Turkey
On separating invariants over prime fields

Separating invariants have been studied extensively in the last decade. They have nice properties that generating sets (as an algebra) do not have, especially in the modular case. Nevertheless, many results on separating invariants require that the underlying field should have sufficiently many elements, which seems to be a very natural condition. In this talk, we will restrict our attention only to the case where the underlying field is prime and discuss the problem of finding separating invariants.

MUFIT SEZER, Bilkent University
Explicit separating invariants for cyclic p-groups

We consider a finite dimensional indecomposable modular representation of a cyclic p-group and we give a recursive description of an associated separating set: We show that a separating set for a representation can be obtained by adding, to a separating set for any subrepresentation, some explicitly defined invariant polynomials.

R. JAMES SHANK, University of Kent, Canterbury, CT2 7NF, United Kingdom
Rings of Invariants and Varieties of Representations

Suppose that G is a finite group, F is a field and V is finite dimensional representation of G over F. The action of G on V induces an action on the dual V^* which extends to an action by algebra automorphisms on the symmetric algebra $S := S(V^*)$. The subring of fixed points, S^G, is known as the ring of invariants of V. For fixed G, F, and $\dim(V)$, the representations of G can be parameterised by an algebraic variety. I will discuss the resulting parameterisation of invariant rings, using modular representations of elementary abelian p-groups as illustrative examples.

DAVID WEHLAU, Royal Military College
Modular Invariant Theory of the Cyclic Group via Classical Invariant Theory

Let C_p denote the cyclic group of prime order p. An important problem in modern invariant theory is to compute (generators for) the ring of invariants of a C_p representation over a field F of characteristic p. Up until now, this has only been done for a few representations.

The central problem in invariant theory in the nineteenth century and early twentieth century was to compute (generators for) the ring of invariants of a complex representation of $\mathrm{SL}_2(C)$. Up to one third of the algebra papers published in the 1880's concerned this problem.

In this talk I will describe a surprising connection between these two problems and my recent result which demonstrates that the two problems are equivalent. Using this we are able to use classical results to give generators for many new modular representations of C_p.