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Locally nilpotent derivations of rings with roots adjoined

We study $\mathbb{G}_a$-actions on certain affine varieties V which admit a dominant morphism to affine space $\mathbb{A}^n$. In particular, V is a variety whose coordinate ring B is the simple algebraic extension of the polynomial ring R defined by $B = R[z]$, where $z^n \in R$ for $n \geq 2$. In many cases, our methods show either that V admits no non-trivial $\mathbb{G}_a$-actions, or that the only $\mathbb{Z}_n$-homogeneous actions are those which lift from $\mathbb{A}^n$. Important examples include the Russell cubic threefold $X \subset \mathbb{A}^4$, defined by $x + x^2y + z^2 + t^3 = 0$, and the Pham–Brieskorn varieties $V \subset \mathbb{A}^n$, defined by $x_1^{e_1} + \cdots + x_n^{e_n} = 0$ for integers $e_i \geq 2$. By studying the locally nilpotent derivations (LNDs) of $B = R[z]$ relative to the affine ring R, we reveal a surprisingly effective general approach to understanding large classes of these varieties. In particular, our work features:

(1) criteria to determine that certain rings are rigid;
(2) a concise new proof that the Russell cubic X is not isomorphic to affine space;
(3) proofs for rigidity for a large family of Pham–Brieskorn threefolds and related varieties; and
(4) numerical criteria to determine when a homogeneous LND of the polynomial ring $A[x_1, \ldots, x_n]$ over A contains a variable x_i in its kernel.

A number of important open problems will also be discussed.

This talk will discuss joint work with Lucy Moser-Jauslin.