
Discrete Geometry
Géométrie discrète
(Org: **Barry Monson** (UNB) and/et **Egon Schulte** (Northeastern))

LEAH BERMAN, University of Alaska Fairbanks, Department of Mathematics & Statistics, PO Box 756660, Fairbanks, AK 99775-6660, USA

Highly Incident Configurations

A geometric (q, k) -configuration is a collection of points and straight lines in the Euclidean (or projective) plane, so that every point lies on q lines and every line passes through k points; if $q = k$ we refer to a k -configuration. We say such a configuration is *highly incident* if $q, k \geq 4$. In this talk, we will discuss a recently discovered infinite class of highly incident $(2s, 2t)$ -configurations with the symmetries of a m -gon, for any $m > 2(q + k - 1)$ (and any $s, t \geq 2$). In particular, this class of configurations includes the only known infinite class of 6-configurations.

KAROLY BEZDEK, University of Calgary, 2500 University Drive NW, Calgary, AB

On a generalization of the Blaschke–Lebesgue theorem

We prove an extension of the Blaschke–Lebesgue theorem for a family of convex domains including disk-polygons. In this way we obtain another new proof for the well-known theorem of Blaschke and Lebesgue.

This is a joint work with Mate Bezdek (Calgary).

TED BISZTRICZKY, University of Calgary, Calgary, Alberta

Separation in neighbourly convex 4-polytopes

Let O be any interior point of a neighbourly 4-polytope P . The Separation Problem concerns the minimum number k of hyperplanes (in real 4-space) that are sufficient to separate O from any facet of P . The conjecture (due to I. Gohberg, H. Hadwiger, A. Markus, and reformulated by K. Bezdek) is that $k < 16$.

We present a survey of progress on this problem in the last twenty years, and present recent results that are joint work with F. Fodor and D. Oliveros.

ANA BREDA, University of Aveiro, Campus de Santiago, 3810-193, Aveiro, Portugal

Deformations of Spherical Isometric Foldings

Given two spherical isometric foldings f and g , f is said to be *deformable* in g if and only if there exists a continuous homotopy $H: [0, 1] \times S^2 \rightarrow S^2$, such that for each $t \in [0, 1]$, H_t given by $H_t(x) = H(t, x)$ is an isometric folding.

The deformation of special classes of isometric foldings will be considered. It will be shown that, within these classes, any isometric folding is continuously deformable in the *standard spherical folding* f_* ($f_*(x, y, z) = (x, y, |z|)$), reinforcing Robertson's conjecture.

Supported partially by CIDMA, through the Foundation for Science and Technology (FCT).

ANTONIO BREDA, University of Aveiro, Aveiro, Portugal

Stretching chirality to the extreme

The phenomenon of chirality appears in many areas of science with special incidence in chemistry and dramatic consequences in life sciences. It is common in several branches of mathematics such as topology, geometry and combinatorics. Chirality of regular oriented "objects" like maps, hypermaps and polytopes is not merely a binary invariant but can be quantified by

two invariants—the chirality group and the chirality index. While every finite abelian group arises as a chirality group of some (hyper)map, many non-abelian groups, including symmetric and dihedral groups, cannot arise as chirality groups. The most extreme type of chirality arises when the chirality group coincides with the monodromy group. Such (hyper)maps are called totally chiral and they seem to be extremely rare. Examples of them though can be constructed by considering appropriate "asymmetric" pairs of generators of certain non-abelian simple groups. In this talk we speak about chirality in maps and hypermaps giving more emphasis to the extreme case of chirality.

ROBERT DAWSON, Saint Mary's University, Halifax, NS

Some Chebyshev Sets in Hyperspaces

A set in a metric space has the "Chebyshev property" if the metric projection function is well defined; that is, if every point has a unique nearest neighbour. In Euclidean spaces this is very closely related to convexity; but even in comparatively familiar spaces such as the "taxicab plane" neither property implies the other.

By a hyperspace over a metric space we understand a collection of compact sets with the Hausdorff metric. There are various apparently unrelated types of Chebyshev set in such hyperspaces; we will examine some of these and look at some results and conjectures that might lead to a complete characterization.

WENDY FINBOW-SINGH, Saint Mary's University, 923 Robie St., Halifax, NS B3H 3C3

Isostatic Almost Spherical Frameworks via Disc Decomposition

We investigate a class of graphs formed from triangulated spheres by removing some edges of the triangulated sphere to form holes, and inserting edges elsewhere between vertices of the triangulated sphere to form polyhedral blocks. The holes and blocks are created in such a way as to balance the count; that is, the number of edges removed from the holes equals the number of edges added to form blocks. We confirm the rigidity and independence of almost all realizations of the graphs in this class in 3-space.

MARK MIXER, Northeastern University, Boston, MA

Transitivity of Graphs Associated with Highly Symmetric Polytopes

An abstract polytope \mathcal{P} is a partially ordered set of faces with some determining properties. Associated with this partial order are many graphs, including the comparability graph and the Hasse diagram for \mathcal{P} . In this talk, we explore the relationship between automorphisms of subgraphs of these two graphs and automorphisms of \mathcal{P} itself.

BARRY MONSON, University of New Brunswick, Box 4400, Fredericton, NB

The Tomotope

The tomotope \mathcal{T} is an abstract 4-polytope consisting of a tightly knit assembly of four tetrahedra and four hemioctahedra shared among just four vertices (and is so-named as a small present to Tomas Pisanski). \mathcal{T} is the first polytope (convex or abstract) whose monodromy group was found *not* to be a string C -group. I will construct \mathcal{T} and report on my confused understanding of its regular covers.

This is an offshoot of a collaboration with D. Pellicer and G. Williams.

JOY MORRIS, University of Lethbridge, 4401 University Dr., Lethbridge, AB

The Cayley Isomorphism Problem

Finding alternate representations of a fixed graph as a Cayley graph can be useful in determining embeddings of the graph onto surfaces, amongst other applications.

It is easy to see that if α is an automorphism of the group G , then the Cayley graph $\text{Cay}(G; S)$ is isomorphic to the Cayley graph $\text{Cay}(G; \alpha(S))$.

A group G has the CI-property if this is the only way to obtain two Cayley graphs on G that are isomorphic. More precisely, G has the CI-property if whenever $\text{Cay}(G; S)$ is isomorphic to $\text{Cay}(G; T)$, there is a group automorphism β of G , such that $\beta(S) = T$. The CI-problem is the problem of determining which groups have the CI-property.

I will present an overview of the CI-problem, including some recent developments and open problems.

ELISSA ROSS, York University, 4700 Keele Street, Toronto, ON M3J 1P3

The Rigidity of Graphs on a Flexible Torus

Taking motivation from the study of the molecular structure of zeolites, we consider the rigidity properties of infinite periodic frameworks. We can think of such a framework in n dimensions as a multigraph embedded on an n -dimensional torus, where the torus may be of fixed or variable dimensions. We use the language of gain graphs to describe this embedding, and we aim to characterize the generic infinitesimal rigidity of infinite periodic frameworks by the properties of the underlying (finite) gain graph.

This talk will outline new results characterizing the rigidity of graphs on the flexible torus, which builds on earlier work about the rigidity of such graphs on the torus with fixed dimensions.

EGON SCHULTE, Northeastern University, Department of Mathematics, Boston, MA 02115, USA

Parasite Constructions for Chiral Polytopes

Chiral polytopes are abstract polytopes with maximum rotational combinatorial symmetry. Their automorphism groups have two flag-orbits represented by pairs of adjacent flags. Regular polytopes, which are characterized by maximum combinatorial symmetry by reflection, have been well-studied, and much work has been done on their classification and groups. By contrast, relatively little is known about abstract chirality of polytopes. We report about recent progress in this area.

ASIA WEISS, York University

Uniform maps on surfaces of non-negative Euler characteristic

We classify uniform (vertex-transitive) polyhedral maps on surfaces of non-negative Euler characteristic and show that all of them have at most 10 orbits on flags. Furthermore we show that they are all quotients of maps obtained by certain operations applied to regular tessellations on the sphere and on the Euclidean plane.

WALTER WHITELEY, York University, Toronto, Ontario

Transfer of infinitesimal and finite rigidity among metrics

It is common to speak about shared properties, and differences, among Euclidean, spherical and hyperbolic spaces of the same dimension. In rigidity, it has been recognized for some time that infinitesimal rigidity or flexibility of a framework $G(p)$ can be transferred between Euclidean \mathbb{E}^d , spherical \mathbb{S}^d and hyperbolic \mathbb{H}^d spaces with the same underlying projective configuration P . We show some simple matrix operations (multiplication by invertible matrices) which carry out this transfer, and we will include Minkowski metric \mathbb{M}^d in the mix.

This transfer for infinitesimal rigidity raises the question of when finite rigidity, and global rigidity, also transfer. We describe some joint work with Bernd Schulze, based on symmetry, which provides transfer finite flexibility of a framework among the metrics. We also mention some results from work with Robert Connelly showing the transfer of generic global rigidity of a graph G among the same metrics.

All of these transfer results speak to an underlying, shared projective geometric theory of infinitesimal rigidity and static rigidity, and the way this implicit projective foundation threads through the wider theory of rigidity.

This is joint work with Franco Saliola.

GORDON WILLIAMS, University of Alaska Fairbanks
Minimal Regular Covers of the Archimedean Tilings

In 1999, M. I. Hartley showed that all abstract polytopes may be represented as quotients of regular abstract polytopes. In this talk we will explore some of the special problems and surprising insights that may be obtained about the structure of such quotients by considering the special case of the uniform tilings of the plane. In particular, we will discuss the progress that has been made on finding *minimal* covers for these tilings, specifically those covers in which the quotient is determined by the stabilizer subgroup of a flag of the polytope.