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**Algebraic Combinatorics**  
**Combinatoire algébrique**  
(Org: **Li Li** and/et **Alex Yong** (Illinois - Urbana-Champaign))

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**ANDREW BERGET**, Mathematical Sciences Building, One Shields Avenue, University of California, Davis, CA 95616, USA  
*On the rank partition of a matroid*

The rank partition measures how close a matroid is to being a union of bases. I will discuss how the rank partition is nicely reflected in decompositions of matroid base polytopes. There is a proof of this that uses representation theory and degenerations of torus orbits in Grassmannians. I will talk about what this proof suggests about representation generated by decomposable tensors.

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**IAN GOULDEN**, University of Waterloo, Waterloo, Ontario N2T 1R3  
*Maps, Branched Covers and the KP Hierarchy*

Maps in an orientable surface of arbitrary genus and branched covers of the sphere can both be represented by factorizations in the symmetric group, in which the subgroup generated by the factors acts transitively on the underlying symbols (these are called “transitive factorizations”). The generating series for a large class of transitive factorizations satisfies the KP hierarchy. We shall discuss the KP hierarchy and a new algebraic combinatorial proof of the fundamental result that relates Schur function expansions of a series and the Plücker relations. As an application, we give a recurrence for triangulations of a surface of arbitrary genus obtained from the simplest partial differential equation in the KP hierarchy. The recurrence is very simple, but we do not know a combinatorial interpretation of it, yet it leads to precise asymptotics for the number of triangulations with  $n$  edges, in a surface of genus  $g$ .

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**CHRISTOPHE HOHLWEG**, Université du Québec à Montréal  
*Geometry of generalized associahedra*

In this talk, we will consider polytopal realizations of generalized associahedra that are obtained from a permutahedron of a finite Coxeter group. Generalized associahedra are fans which encode the combinatorics and geometry of finite type cluster algebras. In the case of our constructions, the link is made via the theory of Cambrian fans. More precisely, we will discuss recent results and open problems regarding these realizations. Our motivation is to refine our understanding of the combinatorial and geometric properties of finite Coxeter groups that are carried through the construction and may be applied to the study generalized associahedra.

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**KURT LUOTO**, University of British Columbia, Vancouver, BC  
*Quasisymmetric and noncommutative Schur functions*

In recent work, Haglund, Mason, van Willigenburg, and this author introduced a family of quasisymmetric functions which we call quasisymmetric Schur (QS) functions. These naturally refine the (symmetric) Schur functions and form a  $\mathbb{Z}$ -basis of  $\text{QSym}$ , the quasisymmetric function algebra. We showed that this basis has interesting properties such as a Littlewood–Richardson rule for the product of a symmetric Schur with a QS function.

We extend the definition of QS functions to skew QS functions, which are counterparts to the classical skew Schur functions. Intimately related to these are the duals of the QS functions, which form a  $\mathbb{Z}$ -basis of  $\text{NSym}$ , the graded Hopf algebra which is dual to  $\text{QSym}$ . The dual QS functions are noncommutative analogs of the classical Schur functions, having properties such as a Littlewood–Richardson rule and relationship to a poset of compositions which is analogous to Young’s lattice of partitions.

This is joint work with Christine Bessenrodt and Stephanie van Willigenburg.

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**LEONARDO MIHALCEA**, Baylor University, Waco, TX 76798, USA

*Double Schubert polynomials for classical groups*

A classical problem in Schubert Calculus is to find polynomial representatives for (equivariant) Schubert classes in the flag manifolds. In type A, Lascoux and Schutzenberger's (double) Schubert polynomials are canonical such representatives. The situation is more subtle in the other classical types. Using Schur's  $P$  and  $Q$  functions, Billey and Haiman constructed a canonical family of polynomials, which are solutions of certain divided-difference equations. In joint work with T. Ikeda and H. Naruse we use localization techniques, and the factorial  $P$  and  $Q$ -Schur functions of Ivanov, to extend Billey and Haiman's construction to equivariant cohomology. The resulting polynomials possess quite pleasant combinatorial properties: stability, positivity, symmetry.

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**GREGG MUSIKER**, MIT, 77 Massachusetts Ave, Office 2-336, Cambridge, MA 02139

*Linear Systems on Tropical Curves*

A tropical curve  $\Gamma$  is a metric graph with possibly unbounded edges, and tropical rational functions are continuous piecewise linear functions with integer slopes. We define the complete linear system  $|D|$  of a divisor  $D$  on a tropical curve  $\Gamma$  analogously to the classical counterpart. Due to work of Matt Baker and Serguei Norine, there is a rank function  $r(D)$  on such linear systems, as well a canonical divisor  $K$ . Completely analogous to the classical case, this rank function satisfies Riemann–Roch and analogues of Riemann–Hurwitz.

This talk will describe joint work with Josephine Yu and Christian Haase investigating the structure of  $|D|$  as a cell complex. We show that such linear systems are quotients of tropical modules and finitely generated by the vertices of the associated cell complex. Using a finite set of generators,  $|D|$  defines a map from  $\Gamma$  to a tropical projective space, and the tropical convex hull of the image realizes the linear system  $|D|$  as a polyhedral complex.

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**NATHAN READING**, North Carolina State University

*Coarsening polyhedral complexes*

A polyhedron is an intersection of finitely many halfspaces. A polyhedral complex is a finite collection of polyhedra which intersect “nicely”. The support of a polyhedral complex is the union of all of its polyhedra. A polyhedral complex  $C'$  is said to coarsen another complex  $C$  if every polyhedron in  $C'$  is a union of polyhedra in  $C$ .

I will describe a local codimension-2 criterion which characterizes coarsenings of a polyhedral complex with convex support. The criterion broadly generalizes a result of Morton, Pachter, Shiu, Sturmfels and Wienand, which identifies the “semigraphoids” of nonparametric statistics with coarsenings of the polyhedral complex (fan) defined by the braid arrangement. The criterion is also closely related to my earlier work on fans defined by lattice congruences of the weak order. I will sketch the proof, which makes use of a generalization of Tits' solution to the Word Problem and a surprising shortcut for checking whether a set of polyhedra is a polyhedral complex. Time allowing, I will discuss a byproduct of the proof: Given a union  $U$  of polyhedra such that the interior of  $U$  is connected, I give a local criterion for deciding when  $U$  is convex.

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**LUIS SERRANO**, University of Michigan, Ann Arbor

*Cyclic sieving for longest reduced words in the hyperoctahedral group*

We show that the set  $R(w_0)$  of reduced expressions for the longest element in the hyperoctahedral group exhibits the cyclic sieving phenomenon. More specifically,  $R(w_0)$  possesses a natural cyclic action given by moving the first letter of a word to the end, and we show that the orbit structure of this action is encoded by the generating function for the major index on  $R(w_0)$ .

This is joint work with T. Kyle Petersen.

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**KELLI TALASKA**, University of Michigan

*Matrix factorization and path enumeration*

Classical work on total positivity in matrices and recent results on total positivity in Grassmannians give us ways to factor totally positive matrices into products of elementary Jacobi matrices. We can represent and manipulate these factorizations using planar networks. This talk will explore how to extend some of these results to matrices which are not necessarily totally positive.

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**PETER TINGLEY**, Massachusetts Institute of Technology  
*Universal Verma modules and the Misra–Miwa Fock space*

The Misra–Miwa  $v$ -deformed Fock space is a representation of the quantized affine algebra of type  $A$ . It has a standard basis indexed by partitions, and the non-zero matrix entries of the action of the Chevalley generators with respect to this basis are powers of  $v$ . Partitions also index the polynomial Weyl modules for the integral quantum group associated to  $\mathfrak{gl}(N)$ , as  $N$  tends to infinity. We explain how the powers of  $v$  which appear in the Misra–Miwa Fock space also appear naturally in the context of Weyl modules. The main tool we use is the Shapovalov determinant for a universal Verma module.

This is joint work with Arun Ram.

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**HENNING ULFARSSON**, Reykjavik University, Menntavegi 1, 101 Reykjavik, Iceland  
*Equivalence relations on permutations and pattern avoidance*

Usually when one studies pattern avoidance of permutations one fixes a particular pattern and counts the permutations that avoid the pattern. In this talk we will study the same counting problem when permutations are placed into equivalence classes with respect to some relation. Then the sizes of the classes that entirely avoid a pattern are added up. This leads to some new and interesting counts.

Three particular relations will be discussed, conjugacy, Knuth equivalence and toric equivalence. The first one allows one to identify the permutations that do not contain a cycle of a prescribed length; the second one provides a new proof of the count for permutations avoiding 231 and 213; and the last one gives some surprising connections with number theory.

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**DAVID WAGNER**, University of Waterloo, Waterloo, Ontario N2L 3G1  
*The lattice of integer flows of a regular matroid*

For a finite multigraph  $G$ , let  $\Lambda(G)$  denote the lattice of integer flows of  $G$ —this is a finitely generated free abelian group with an integer-valued positive definite bilinear form. Bacher, de la Harpe, and Nagnibeda show that if  $G$  and  $H$  are 2-isomorphic graphs then  $\Lambda(G)$  and  $\Lambda(H)$  are isometric, and remark that they were unable to find a pair of nonisomorphic 3-connected graphs for which the corresponding lattices are isometric. We explain this by examining the lattice  $\Lambda(M)$  of integer flows of any regular matroid  $M$ . Let  $M_\bullet$  be the minor of  $M$  obtained by contracting all co-loops. We show that  $\Lambda(M)$  and  $\Lambda(N)$  are isometric if and only if  $M_\bullet$  and  $N_\bullet$  are isomorphic.