Operator Algebras
Algèbres d'opérateurs

(Org: **Andrew J. Dean** (Lakehead), **George Elliott** (Toronto) and/et **Marco Merkli** (Memorial))

SERBAN BELINSCHI, University of Saskatchewan, 106 Wiggins Rd, Saskatoon, SK, S7N 5E6

Free infinite divisibility for the classical normal distribution

In this talk we will show that the classical Gaussian is infinitely divisible with respect to *free* additive convolution; this is a rather surprising result, as this distribution played a minor role in free probability. We will give a combinatorial background and motivation for this investigation, and show the main points of the analytic proof of the result.

This is joint work with M. Bożejko, F. Lehner and R. Speicher.

MAN-DUEN CHOI, University of Toronto, 40 St. George St., Toronto, Ontario, M5S 2E4

What sort of non-commutative analysis is needed in quantum computing?

Suddenly, the era of quantum computers arrives. We need matrix analysis to tackle some concrete problems in the confusing world of non-commutative geometry and non-commutative probability.

KEN DAVIDSON, University of Waterloo, Waterloo, ON, N2L 3G1

Semicrossed products of the non-commutative disc algebra

The non-commutative disk algebra $\mathfrak{A}_n$ is the norm closed nonself-adjoint operator algebra generated by n isometries with pairwise orthogonal ranges. The automorphisms are determined by conformal automorphisms of the unit ball in $\mathbb{C}^n$. I will discuss the classification and representation theory of the semi-crossed product of $\mathfrak{A}_n$ by an automorphism.

ANDREW J. DEAN, Lakehead University, 955 Oliver Road, Thunder Bay, Ontario, P7B 5E1

Classification of C^*-dynamical systems

We shall discuss recent progress on developing invariants to classify C^*-dynamical systems.

GEORGE ELLIOTT, University of Toronto

Some remarks on classification

Some remarks will be made concerning the functorial aspects of classification. For instance, the Connes–Takesaki flow of weights, known to be of critical importance as an invariant for von Neumann algebras, and to be functorial at the level of isomorphisms, will be shown still to be functorial when certain more general, although still rather special, homomorphisms are considered as well. This functor is then full, when the category of flows is considered with arbitrary homomorphisms.

CLAUS KOESTLER, St. Lawrence University

Noncommutative independence and characters of the infinite symmetric group

Thoma's theorem identifies the extremal characters of the infinite symmetric group. We give an alternative proof of this characterization result based on noncommutative independence as it emerges from exchangeability through noncommutative de Finetti theorems.

This is joint work with Rolf Gohm.

GREG MALONEY, University of Toronto, 27 King's College Circle, Toronto, ON, M5S 1A1
Tilings, continued fractions, and C^*-algebras

Ten years ago, J. Mingo classified, up to translation, all one-dimensional tilings arising from the projection method applied to a line in two-dimensional space. The invariant, which is closely related to the continued fraction expansion of the slope of the line, is a sequence space with an equivalence relation. The associated quotient topological space is difficult to study because it is not Hausdorff (in fact, every equivalence class is dense), but there is an associated AF C^*-algebra that has an easily computable Bratteli diagram.

In this talk I will describe how this sequence space and its associated C^*-algebra can be constructed for projection tilings of a line in higher-dimensional space. A generalization of the usual continued fraction expansion is central to this construction; however, whereas in two dimensions the continued fraction expansion of the slope of a line is essentially unique, in higher dimensions there are many different continued fraction expansions with different convergence properties. Hence it is the continued fraction, not the line, that determines the sequence space and C^*-algebra. I will describe some properties of the continued fraction that can be formulated in terms of K-theory.

GORDON MCDONALD, UPEI

MARCO MERKLI, MUN
Repeated interaction quantum systems

We consider a quantum system $\mathcal{S}$ interacting successively with elements $\mathcal{E}$ of an infinite chain $\mathcal{C} = \mathcal{E} + \mathcal{E} \cdots$. Each interaction is characterized by an interaction time τ and an interaction operator V acting on $\mathcal{S}$ and one of the $\mathcal{E}$. We show that such systems reach an asymptotic state as time tends to infinity. We construct the latter state explicitly (by perturbation theory), linking it to the spectral data of an effective reduced dynamics operator. We explain the physical (thermodynamic) properties of such asymptotic states.

The mathematical framework is that of algebraic quantum (field) theory. We use the Tomita–Takesaki structure to represent the dynamics in a suitable manner by powers of a reduced dynamics operator. The long-time asymptotics is determined by spectral information of that operator, and is analyzed by perturbation theory in the coupling between $\mathcal{S}$ and $\mathcal{C}$.

This is joint work with Laurent Bruneau (Cergy–Pontoise) and Alain Joye (Institut Fourier).

JAMES A. MINGO, Queen's University, Kingston, Ontario, K7L 3N6
Higher order freeness

Second order freeness was invented by Roland Speicher and myself to do for fluctuations of random matrices what Voiculescu did for moments. Namely if one has two ensembles of random matrices X_N and Y_N and one knows their limiting distributions and that they are asymptotically free, then one knows the limiting distributions of sums and products of X_N and Y_N. If they are asymtotically free of second order then one can do the same for their fluctuations. The idea of first and second order freeness can be extended to freeness of all orders. I will discuss recent developments and problems in higher order freeness.

FERNANDO MORTARI, University of Toronto, Toronto, Ontario
Tracial state spaces of higher stable rank simple C^*-algebras

The first examples of simple C^*-algebras with stable rank other than one or infinity were given by Villadsen, 10 years ago. Since then, other examples have been found (Toms); all of the existing examples admit a unique tracial state. It is therefore natural to ask whether examples can be found of simple C^*-algebras with higher stable rank (including infinity) and more than one tracial state. I'll describe examples of such algebras that admit arbitrary tracial state spaces, and some of the difficulties involved in constructing these examples.

MATTHIAS NEUFANG, Fields Institute, 222 College Street, Toronto, Ontario, M5T 3J1
The intrinsic group of a locally compact quantum group

Locally compact quantum groups (LCQGs), as introduced and studied by Kustermans–Vaes in 2000, provide a category which comprises both classical group algebras and group-like objects arising in mathematical physics such as Woronowicz's famous quantum group $\mathrm{SU}_q(2)$. In joint work with M. Junge and Z.-J. Ruan, we have investigated the quantum group analogue of the class of completely bounded multiplier algebras which play an important role in Fourier analysis over groups, by means of a representation theorem. This result yields an interesting class of quantum channels, and enables us to express quantum group duality precisely in terms of a commutation relation for these channels. In this talk, I shall mainly focus on very recent work with my Ph.D. student M. Kalantar which shows that those quantum channels with noiseless error correction can be identified with the intrinsic group of the dual quantum group. We thus assign, to each LCQG, a locally compact group that is an invariant for the latter; this functor preserves compactness as well as discreteness (hence also finiteness), and, for large classes of quantum groups, amenability. For Woronowicz's class of compact matrix pseudogroups, we always obtain a compact Lie group—which in the case of $\mathrm{SU}_q(2)$ is precisely the circle group. Combining our construction with the above-mentioned commutation result, we can further assign, to each LCQG, a certain subgroup of the circle group that forms a numerical invariant generalizing Heisenberg's bicharacters, and may be used towards a classification of LCQGs.

ZHUANG NIU, University of Oregon
Lifting KK-elements, asymptotical unitary equivalence and the classification of simple C^*-algebras

Two existence theorems concerning KK-elements and rotation maps are proved. More precisely, let A and B be simple unital AH-algebras (with slow dimension growth) of real rank zero. Then any positive element in $KK(A,B)$ can be lifted to a homomorphism from A to B. Moreover, if A is a unital subalgebra of B, the embedding can be perturbed by an approximated inner automorphism of B to realized any given rotation map, but keep the induced KK-element unchanged.

These two existence theorems were used in the classification of simple C^*-algebras which are tracially AF after tensoring a UHF algebra.

This is a joint work with Huaxin Lin.

VLADIMIR PESTOV, University of Ottawa, Department of Mathematics and Statistics, 585 King Edward Avenue, Ottawa, ON, K1N 6N5
Non-isomorphic C^* algebras with isomorphic unitary groups

Suppose that A and B are two C^*-algebras whose unitary groups $U(A)$ and $U(B)$ are isomorphic (as abstract groups say). Under what conditions are A and B isomorphic? Research in this direction spans half a century, with first results being published by Dye in 1954 and the most recent ones belong to T. Giordano and his pupils A. Booth and A. Al-Rawashdeh. Typically, such results state that the answer is positive under some simplicity-type assumptions on A and B. It seems however that up until recently there were no counter-examples in this direction. What could be the first example of non-isomorphic C^*-algebras A and B whose unitary groups are isomorphic (even as topological groups with the uniform topology) was obtained back in 2003 by this speaker, in his examiner's report on Al-Rawashdeh's Ph.D. thesis, on the basis of the classical Milyutin's theorem about Banach spaces of continuous functions on compacta: namely, it is enough to consider $A = C[0,1]$ and $B = C([0,1]^2)$. Recently, motivated by a conversation with Giordano, this speaker came up with further (commutative) examples in this direction, linked to research on equivalence relations between topological spaces determined by isomorphism of their free abelian topological groups and function spaces in pointwise topology, which was active in Moscow during this speaker's years in graduate school. Obstacles to constructing genuinely non-commutative examples will be also discussed.

BAHRAM RANGIPOUR, University of New Brunswick
Index Theory, past, present, and future

We review the Connes–Moscovici local index formula from Hopf cyclic cohomology point of view. We associate to each Lie–

Cartan pseudogroup a noncommuative-noncocommutative Hopf algebra which is responsible for the local index formula of the geometry invariant under the pseudogroup. We develop appropriated apparatuses for computing the Hopf cyclic cohomology of the Hopf algebras. Finally we discuss some open problems.

LEONEL ROBERT, York University, N520 Ross Building, 4700 Keele Street, Toronto, ON, M3J 1P3
Classification of homomorphisms from $C_0(0,1]$ to a C^*-algebra

I will describe a class of C^*-algebras for which the homomorphisms from $C_0(0,1]$ to the algebra can be classified by means of the Cuntz semigroup functor. I will discuss examples (simple and non-simple) of C^*-algebras for which this classification fails, and some ideas of how to deal with them.

This is joint work with Luis Santiago.

LUIS SANTIAGO, Fields Institute, 222 College Street, Toronto, ON, M5T 3J1
Classification of non-simple C^*-algebras using the Cuntz semigroup

We show that non-simple inductive limits of tree algebras are classified by their Cuntz semigroup. More generally, using the Cuntz functor we classified $*$-homomorphisms—up to approximate unitary equivalence—from inductive limits of tree algebras to a C^*-algebra of stable rank one.

MICHAEL SIGAL, University of Toronto, Toronto, Ontario, M5S 2E4
On quantum, stationary, non-equilibrium states

In this talk I describe recent results (jointly with Marco Merkli and Matthias Mueck) on existence and dynamical stability of stationary, non-equilibrium states in certain models of quantum statistical mechanics.

KEITH TAYLOR, Dalhousie University, Halifax, NS, B3H 3J5
Wavelets with Crystal Symmetries

In [1], Baggett et al. introduced a general framework for multiresolutions and wavelets in a Hilbert space. Given a triple (H,Γ,δ), where H is a Hilbert space, Γ is a discrete group of unitary operators on H and δ is another unitary operator on H for which $\delta^{-1}\gamma\delta$ is an element of Γ for every $\gamma\in\Gamma$, they define both multiresolutions and wavelets. We investigate this situation when $H=L^2(R^2)$, Γ comes from the natural action of a two-dimensional crystal group (wallpaper group) and δ is defined by a dilation matrix compatible with the crystal group. The structure of the C^*-algebra of the crystal group appears to be useful.

Joint work with Josh MacArthur.

References

[1] Larry Baggett, Alan Carey, William Moran and Peter Ohring, *General existence theorems for orthonormal wavelets, an abstract approach.* Publ. Res. Inst. Math. Sci. **31**(1995), 95–111.

AARON TIKUISIS, University of Toronto, Toronto, Ontario
The Cuntz Semigroup for Commutative C^*-algebras

The Cuntz semigroup is an invariant which promises to be important to the classification of C^*-algebras. It is still far from being well-understood—in fact, its computation has only been carried out in some fairly restricted situations. In my talk, I will report on attempts to understand, and even compute, this invariant for certain commutative C^*-algebras. We have a

description of the Cuntz semigroup for the situation that the algebra is separable and the spectrum has dimension at most three.

In defining the Cuntz semigroup of A, a pre-order relation is imposed on the elements of $\bigcup M_n \otimes A$; the relation has been described (informally) Murray–von Neumann equivalence of the support projections. In our study of the Cuntz semigroup when A is commutative, we have taken this description seriously, thus using open projections to represent Cuntz elements.

This is joint work with Leonel Robert.

MARIA GRAZIA VIOLA, Lakehead University, 1 Colborne Street W., Orillia, ON, L3V 7X5

A simple, separable, exact C^*-algebra non-isomorphic to its opposite algebra

It was an open question for a while if there existed a simple, separable, exact C^*-algebra which was not isomorphic to its opposite algebra. In a recent work with C. Phillips we gave an example of such a C^*-algebra A and showed that it has the following additional properties. It is stably finite, approximately divisible, has real rank zero and stable rank one, and has a unique tracial state. Moreover, the order on projections over D is determined by this unique trace, and the C^*-algebra tensorially absorbs the Jiang–Su algebra Z, and the 3^∞ UHF algebra. We could also explicitly compute the K-theory of D, namely $K_0(D) \cong \mathbb{Z}[\frac{1}{3}]$ with the standard order, and $K_1(D) = 0$. Some open questions about simple, separable C^*-algebras with some additional properties which are not isomorphic to their opposite algebras will also be discussed.

DILIAN YANG, University of Windsor, 401 Sunset Avenue, Windsor, ON, N9B 3P4

Endomorphisms and Modular Theory of 2-Graph C^*-Algebras

We consider endomorphisms and modular theory of the graph C^*-algebra of a 2-graph on a single vertex. We prove that there is a semigroup isomorphism between unital endomorphisms of a 2-graph C^*-algebra and its unitary pairs with a twisted property. We characterize when endomorphisms or automorphisms preserve the fixed point algebra of gauge automorphisms and its canonical masa. Some other properties of endomorphisms are also investigated.

As far as the modular theory is concerned, we show that the algebraic *-algebra generated by the generators of a 2-graph C^*-algebra with the inner product induced from a distinguished state is a modular Hilbert algebra. We will also discuss the type of the von Neumann algebra generated by its GNS representation.

BEATRIZ ZAMORA-AVILES, York University

Analytic P-ideals of $\mathcal{B}(H)^+_{\leq 1}$

Let H be a separable infinite dimensional complex Hilbert space and $\mathcal{B}(H)^+_{\leq 1}$ the set of positive operators on H of norm at most one. Considering $\mathcal{B}(H)^+_{\leq 1}$ as a Polish space with respect to the weak operator topology, we prove that certain analytic subsets of $\mathcal{B}(H)^+_{\leq 1}$ (that we call analytic P-ideals) are determined by lower semicontinuous submeasures on $\mathcal{B}(H)^+_{\leq 1}$. This result generalizes a theorem of S. Solecki for the poset $\mathcal{P}(\omega)/\operatorname{Fin}$.