
VLADIMIR PESTOV, University of Ottawa, Department of Mathematics and Statistics, 585 King Edward Avenue, Ottawa, ON, K1N 6N5

Non-isomorphic C^ algebras with isomorphic unitary groups*

Suppose that A and B are two C^* -algebras whose unitary groups $U(A)$ and $U(B)$ are isomorphic (as abstract groups say). Under what conditions are A and B isomorphic? Research in this direction spans half a century, with first results being published by Dye in 1954 and the most recent ones belong to T. Giordano and his pupils A. Booth and A. Al-Rawashdeh. Typically, such results state that the answer is positive under some simplicity-type assumptions on A and B . It seems however that up until recently there were no counter-examples in this direction. What could be the first example of non-isomorphic C^* -algebras A and B whose unitary groups are isomorphic (even as topological groups with the uniform topology) was obtained back in 2003 by this speaker, in his examiner's report on Al-Rawashdeh's Ph.D. thesis, on the basis of the classical Milyutin's theorem about Banach spaces of continuous functions on compacta: namely, it is enough to consider $A = C[0, 1]$ and $B = C([0, 1]^2)$. Recently, motivated by a conversation with Giordano, this speaker came up with further (commutative) examples in this direction, linked to research on equivalence relations between topological spaces determined by isomorphism of their free abelian topological groups and function spaces in pointwise topology, which was active in Moscow during this speaker's years in graduate school. Obstacles to constructing genuinely non-commutative examples will be also discussed.