RAUNO AULASKARI, University of Joensuu, Department of Physics and Mathematics, P.O. Box 111, 80101 Joensuu, Finland

A non-α-normal function whose derivative has finite area integral of order less than $2/\alpha$

Let $\mathcal{D}$ be the unit disk $\{z : |z| < 1\}$ in the complex plane. A function f, meromorphic in $\mathcal{D}$, is normal, denoted by $f \in N$, if $\sup_{z \in \mathcal{D}}(1-|z|^2)f^{\#}(z) < \infty$, where $f^{\#}(z) = |f'(z)|/\big(1+|f(z)|^2\big)$. For $\alpha > 1$, a meromorphic function f is called α-normal if $\sup_{z \in \mathcal{D}}(1-|z|^2)^{\alpha} f^{\#}(z) < \infty$. H. Allen and C. Belna [1] have proved that there is an analytic function f_1, defined in $\mathcal{D}$, such that

$$\iint_{\mathcal{D}} |f_1'(z)|\, dx\, dy < \infty$$

but $f_1 \notin N$. S. Yamashita [3] sharpened this result by showing that for another analytic function f_2 which does not belong to N it holds

$$\iint_{\mathcal{D}} |f_2'(z)|^p\, dx\, dy < \infty \tag{1}$$

for all p, $0 < p < 2$. Further, H. Wulan [2] studied more the function f_2 and showed that $f_2 \notin \bigcup_{0<p<\infty} Q_p^{\#}$ but $f_2 \in \bigcap_{0<p<\infty} M_p^{\#}$. We construct a class of analytic functions f_s which satisfy (1) for $0 < p < \frac{2}{\alpha}$ but $f_s \notin N^{\alpha}$ for $\alpha > 1$. Further, the question if f_s belongs or not to $\bigcup_{0<p<\infty} M_p^{\#}$ is considered.

References

[1] H. Allen and C. Belna, *Non-normal functions $f(z)$ with $\iint_{|z|<1} |f'(z)|\, dx\, dy < \infty$.* J. Math. Soc. Japan **24**(1972), 128–132.

[2] H. Wulan, *A non-normal function related Q_p spaces and its applications.* In: Progress in Analysis **I, II**, World Sci. Publ., River Edge, NJ, 2003, 229–234.

[3] S. Yamashita, *A non-normal function whose derivative has finite area integral of order $0 < p < 2$.* Ann. Acad. Sci. Fenn. Ser. Math. **4**(1978/1979), 293–298.