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Specht Problem for G-graded Algebras

Let W be an algebra over a field F of characteristic zero. Let $\mathrm{id}(W)$ be the T-ideal of identities of W, i.e., polynomials in noncommutative variables which vanish upon any evaluation on W. One of the main theorems in PI theory (due to Kemer, 1991) is the solution of the Specht problem, namely that $\mathrm{id}(W)$ is finitely generated as a T-ideal (an ideal of the free algebra $F\langle X\rangle$ is a T-ideal if it is closed under endomorphisms, that is, variables can be replaced by arbitrary polynomials). The main problem in the solution is the case where the algebra W is affine (i.e., finitely generated over F).

In the last two decades people considered G-graded identities on G-graded algebras where G is any group (generalizing ordinary identities where $G = \{1\}$). Here one considers polynomials in noncommutative G-graded variables, that is variables of the form x_g where $g \in G$. Admissible evaluations are those where the variables x_g are replaced only by elements of W_g. A G-graded identity is a polynomial which vanishes upon any admissible evaluation. The Specht problem for G-graded algebras asks whether the T-ideal of G-graded identities is finitely generated as a T-ideal. The case where $G \cong \mathbb{Z}/2\mathbb{Z}$ was known (due to Kemer) and it was essential in reducing the ordinary Specht problem from non-affine to affine algebras.

In these lectures I will present the main steps in the proof of the Specht problem for PI, G-graded affine algebras where G is a finite group (the nonaffine case follows as in the non-graded case). I will explain the obstacles which arise from the fact that the group G may not be abelian. As in the ungraded case the main part is to show that the T-ideal of G-graded identities of an affine algebra W coincides with the T-ideal of G-graded identities of a G-graded finite dimensional algebra A.

It should be emphasized that a G-graded algebra may be G-graded PI (i.e., PI as a G-graded algebra) even if it is non-PI (e.g., take the free algebra W (on more than one variable), G-graded where $G \neq \{e\}$ and $W_g = 0$ for $g \neq e$). The Specht problem remains open for such algebras.

Most of the new results which I will present were obtained jointly with Alexei Kanel Belov. Other results were obtained with Haile, Natapov, Kassel, Giambruno and La Mattina.