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Mean-variance hedging with portfolio constraints and portfolio insurance

We study a problem of mean-variance hedging which includes a general convex constraint, together with “portfolio insurance” in the form of a guaranteed almost-sure lower bound on the wealth at close of trade. We use a conjugate duality approach, the essence of which is to appropriately “perturb” the problem, and calculate concave conjugates in terms of the perturbation in order to construct a Lagrangian function and a dual cost function, together with a set of Kuhn–Tucker optimality relations which effectively characterize the saddle points of the Lagrangian. Existence of a Lagrange multiplier is established subject to a natural Slater-type condition on the terminal-wealth constraint; the Lagrange multiplier comprises an Ito process paired with a member of the adjoint of the space of essentially bounded random variables measurable with respect to the event sigma-algebra at close of trade. The optimality relations are then used to synthesize an optimal portfolio in terms of the Lagrange multiplier.