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On the invariants of non-abelian p -groups

Let $\rho: G \hookrightarrow \text{GL}(V)$ be a representation of a finite group G over a finite field, \mathbb{F} . The main object of study is the invariant ring $\mathbb{F}[V]^G$ where the action of G on V^* is induced by the representation. Describing $\mathbb{F}[V]^G$ is of fundamental importance and when $|G| \notin \mathbb{F}^\times$ only a little is known. There is no analog of Noether bound, depending only on the group order where Noether bound gives the maximum degree of a polynomial in a minimal generating set.

Invariants of upper triangular unipotent groups are known to be polynomials. Since any p -group is contained in some Sylow p -subgroup, without loss of generality (up to a change of basis), any p -groups can be assumed to be a subgroup of upper triangular unipotent matrices. Hence, understanding invariants of p -groups will enable us to understand modular invariants in detail. Cyclic groups of prime power order p^α are studied in the literature, but only a little is known for nonabelian groups.

In this study, we will restrict our attention to finding invariants of a nonabelian p -group of order p^3 and of exponent p . The first such nontrivial representation occurs in dimension 4. The underlying field will be the prime field and $p > 3$.