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Linear Sparsity Groups and Their Polynomial Invariants

We define a sparsity pattern over a field F as a map $\sigma\colon [n]\times[n]\to\mathcal{P}(F)\setminus\{\emptyset\}$ where $\mathcal{P}(F)$ is the power set consisting of all subsets of F. We show that all matrices in $\mathrm{GL}(n,F)$ subordinate to the pattern σ form a group

$$\mathrm{GL}_\sigma(n,F)=\{M=[a_{ij}]_{n\times n}\in\mathrm{GL}(n,F): a_{ij}\in\sigma(i,j)\}$$

if and only if σ satisfies certain completeness conditions. We prove that the ring of invariants of each finite sparsity subgroup is a polynomial algebra. This result generalizes Dickson's Theorem on invariants of $\mathrm{GL}(n,F_q)$ as well as many other examples. By viewing $\mathrm{GL}(n,F)$ as a Lie group we obtain another interpretation of an arbitrary sparsity pattern which can be generalized to Chevalley groups and reductive algebraic groups.