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Floer spheres, small cyclic branched covers and the Kontsevich integral

Finding Floer spheres—irreducible homology spheres with trivial instanton Floer homology $HF_*(Y)$—is an intriguingly difficult problem, stated as Problem 3.106 in Kirby's updated problem list. The Casson invariant $\lambda(Y)$ can be regarded as a first obstruction to their existence. In this lecture, we are interested in the case of cyclic branched covers along knots, where the covering action $\tau \colon Y \to Y$ can be exploited to detect non-trivial actions on Floer homology or essential surfaces in Y. In particular, the 2-loop part of the Kontsevich integral of a knot can then be interpreted as a second obstruction to the existence of Floer spheres in the class of cyclic branched covers along knots. As an application, we obtain non-vanishing results for the Floer homology of various classes of cyclic branched covers which are small 3-manifolds.