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Computing Neumann–Laplacian eigenvalues in nonsmooth domains and applications

The study of shape optimization problems involving the eigenvalues of an elliptic operator (the Laplacian-Δ for instance) has strong relations with several applications (stability of vibrating bodies, ...). When Dirichlet boundary conditions are imposed on the free boundary $\partial\Omega$, there are several situations where solving the optimization problem is made easier, in both theoretical and computational aspects, by the existence of a relaxation process. The same problems for the Laplace operator with Neumann conditions on the free boundary is a difficult subject where up to now very few results are available. In fact, proving existence of solutions in this case is related to the (deep) understanding of the behavior of the Neumann spectrum on non-smooth domains (highly oscillating boundaries, cracks, etc). This is a challenging question and it is largely open.

In this talk, we consider the Neumann–Laplacian eigenvalue problem in domains with multiple cracks. We derive a mixed variational formulation which holds on the whole geometric domain (including the cracks) and implement efficient finite element discretizations for the computation of eigenvalues. Optimal error estimates are given and several numerical examples are presented, confirming the efficiency of the method. As applications, we numerically investigate the behavior of the low eigenvalues of the Neumann–Laplacian in domains with a high number of cracks. For particular cases of highly oscillating boundaries, these computations allows us to identify formally the problems with the limit spectra.