Probability
Probabilités

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JEAN BERTOIN, Université Pierre et Marie Curie, Paris

Allelic partitions for Galton–Watson processes

We consider a (sub) critical Galton–Watson process with neutral mutations (infinite alleles model), and decompose the entire population into clusters of individuals carrying the same allele. We specify the law of this allelic partition in terms of the distribution of the number of clone-children and the number of mutant-children of a typical individual. The approach combines an extension of Harris representation of Galton–Watson processes and a version of the ballot theorem.

FRANCIS COMETS, University Paris Diderot–Paris 7

Billiards on general tables with random reflections

We study stochastic billiards on general tables: a particle moves according to its constant velocity inside some domain D in R^d until it hits the boundary and bounces randomly inside according to some reflection law. We assume that the boundary of the domain is locally Lipschitz and almost everywhere continuously differentiable. The angle of the outgoing velocity with the inner normal vector has a specified, absolutely continuous density. The cosine density is of special interest.

Joint works with Serguei Popov, Gunter Schutz and Marina Vachkovskaia.

DON DAWSON, Carleton University, Ottawa, Canada

Continuum limits of branching systems on the hierarchical lattice

The fact that rescaled limits of some classes of interacting particle systems and interacting diffusions on the Euclidean lattice give rise to super-Brownian motion has been established by a number of authors. In this lecture the corresponding question is considered for branching and catalytic branching systems on the hierarchical lattice and some partial results based based on the hierarchical mean field limit are described.

This is joint work with A. Greven and I. Zähle.

JEAN-FRANÇOIS DELMAS, CERMICS–ENPC, 6 & 8 avenue B. Pascal, 77455 Champs sur Marne, France

General pruning and immigration for continuum random trees

Pruning and immigration for Galton–Watson trees are well known. I will present the analogue for critical or sub-critical continuum random trees (CRT). The pruning mechanism relies on a representation of the CRT based on Lévy snake. After introducing a general pruning procedure for CRT, I will present the dual process of immigration with intensity proportional to the size of the population. This immigration can be studied through the continuous state branching process. This process does not keep track of the genealogy, but it is easier to handle.

The pruning or immigration allow to describe a population undergoing neutral mutations. In the case of extinction in finite time, using a decomposition of Williams type, it is easy to compute the probability for the last individual of the population to be of the same type as its original ancestor.

Then I will recall the link between a particular family of pruning mechanism and some fragmentation processes. This family of pruning yields a nice way to reduce a CRT. Using a Girsanov formula, it is possible to define a super-critical CRT and to extend the pruning to such CRT. I will present another approach for the case of CRT with quadratic branching mechanism, based on reflected Brownian motion with drift.

BENJAMIN DOYON, Durham University, Department of Mathematical Sciences, South Rd, Durham DH1 3LE, United Kingdom
The stress-energy tensor in conformal loop ensembles

The scaling limit of two-dimensional statistical models at criticality is believed to be described by two distinct mathematical frameworks: the algebraic framework of vertex operator algebras—this is what is usually called conformal field theory (CFT), and the probabilistic framework of Schramm–Loewner evolution (SLE) and its generalisations, in particular conformal loop ensembles (CLE). The first one leads to many non-trivial predictions about local observables, but its axioms are based on physically-motivated principles of quantum field theory. The second one deals with random curves and loops and its connection to statistical models can often be proven, but as of yet says little about local observables and the powerful algebraic structure which describes them. The main local observable, at the basis of the algebraic structure, is the stress-energy tensor. After reviewing the basics of CLE for $8/3 < \kappa < 4$, I will propose a definition for the stress-energy tensor which reproduces results from CFT: the conformal Ward identities with non-zero central charge, its transformation property with Schwarzian derivative (leading to the Virasoro algebra), and its relation to small variations of the boundary of the domain where the loops lie. This construction generalizes that of the case of zero central charge (with V. Riva and J. Cardy, 2006), SLE at $\kappa = 8/3$.

ALISON ETHERIDGE, University of Oxford
Evolution in a spatial continuum

Understanding the evolution of individuals which live in a structured and fluctuating environment is of fundamental importance in mathematical population genetics. We outline some of the mathematical challenges that arise from modelling structured populations, primarily focussing on the interplay between forwards in time models for the evolution of the population and backwards in time models for the genealogical trees relating individuals in a sample from that population. In addition to the classical models we describe a new model, introduced in recent work with Nick Barton, which can be thought of as a spatial version of the generalised Fleming–Viot process and for which genealogical trees are spatial versions of Lambda- (or more generally Xi-) coalescents.

ANTAL JARAI, Carleton University, School of Math. & Stat., 1125 Colonel By Drive, Ottawa, ON, Canada, K1S 5B6
Geometric bounds on the Uniform Spanning Forest in high dimensions

We prove volume and resistance estimates for the Uniform Spanning Forest in $\mathbf{Z}^d$ in dimensions $d \geq 5$. We give estimates on the upper and lower tail behaviour of the number of edges in the component of the origin inside a Euclidean ball of radius R, when this quantity is rescaled by R^4. The bounds can be used to study random walk restricted to a component of the Uniform Spanning Forest.

MIKE KOZDRON, University of Regina, College West 307.31, Regina, Saskatchewan S4S 0A2, Canada
Estimates for the diameter of a chordal SLE path

We derive an estimate for the diameter of a chordal SLE path in the upper half plane $\mathbb{H}$ between two real boundary points 0 and $x > 0$. Specifically, we prove that if $\kappa \in (0, 8)$ and $\gamma\colon [0,1] \to \overline{\mathbb{H}}$ is a chordal SLE$_\kappa$ in $\mathbb{H}$ from 0 to x, then $P\{\gamma[0,1] \cap C_R \neq \emptyset\} \asymp R^{1-4a}$ where $a = 2/\kappa$ and C_R denotes the circle of radius Rx centred at 0 in the upper half plane. As an application of our result, we derive an estimate that two nearby points, one on the boundary and one in the interior, are swallowed together by a chordal SLE$_\kappa$ path, $4 < \kappa < 8$.

This talk is based on joint work with Tom Alberts of the Courant Institute.

NEAL MADRAS, York University, 4700 Keele Street, Toronto, Ontario M3J 1P3, Canada
Pattern-Avoiding Permutations: A Probabilist's View

A pattern of length k is a permutation $(a[1], \ldots, a[k])$ of $\{1, \ldots, k\}$. This pattern is said to be contained in a permutation of $\{1, \ldots, N\}$ (for $N > k$) if there is a subsequence of k elements of the (long) permutation that appears in the same relative order as the pattern. (E.g. the pattern (132) is contained in the permutation (6425713) because the permutation contains the subsequence (273).) For a given pattern P, let $A_N[P]$ be the number of permutations of $\{1, \ldots, N\}$ that do not contain P. Combinatorialists have proven that $A_N[P]$ grows exponentially in N (rather than factorially), but little is known about the numerical value of the exponential growth rate. For example, which patterns of length 5 are easiest/hardest to avoid?

After introducing the background, I shall describe some Monte Carlo investigations into this and related problems. The design and implementation of the Monte Carlo method raise some interesting mathematical problems. The results of the Monte Carlo lead to some new conjectures, including a description of what a "typical" 4231-avoiding permutation looks like. Finally, I shall outline some rigorous progress on this last problem. The Monte Carlo work was done by Hailong Liu as an NSERC Undergraduate Student Research Awardee.

GREGORY MIERMONT, Université Pierre et Marie Curie, 175 rue du Chevaleret, 75013 Paris, France
Some geometric aspects of scaling limits of random maps

A map is an embedding of a graph in a surface that cuts the latter into disks. Random maps are used by physicists as a way to sample a discrete random surface, which is supposed to approximate some continuous limit when the mesh of the graph gets small while its size goes to infinity.

This leads to the mathematical problem of convergence in distribution of random maps, considered as a metric space by endowing them with a properly rescaled version of the graph distance, towards some limiting metric space. In this talk, I will focus on some aspects of the scaling limits of maps, like the uniqueness of the typical geodesic path.

PIERRE NOLIN, Ecole Normale Supérieure, 45 rue d'Ulm, 75005 Paris, France
Gradient Percolation and the geometry of diffusion fronts

We discuss a model of inhomogeneous medium, called "Gradient Percolation". This model, first introduced by physicists, is a percolation-type model where the density of occupied sites depends on the location in space. The macroscopic interface (separating occupied sites and vacant sites) that appears was used to model phenomena like diffusion or chemical etching. This interface remains localized in regions where the density of occupied sites is close to the percolation threshold p_c, and its macroscopic behavior is related to near-critical standard percolation. We show in particular that its fluctuations and its length can be described via the critical exponents of standard percolation, and that it is locally asymmetric (on every scale). We finally study a natural two-dimensional model where many particles that start at the origin diffuse independently, that provides a natural example where critical fractal geometries spontaneously arise.

DENIS SEZER, York University
Conditioning super-Brownian motion on its exit measure X_D

Let X be a super-brownian motion defined on a domain E in the euclidean space and (X_D) be its exit measures indexed by sub-domains of E. We pick a sub-domain D and condition the super-brownian motion inside this domain on its exit measure X_D. We give an explicit construction of the resulting conditional law in terms of a particle system, which we call the "backbone", along which a mass is created uniformly. In the backbone, each particle is assigned a measure ν at its birth. The spatial motion of the particle is an h-transform of Brownian motion, where h is a potential that depends on both ν and the particle's birth location. ν represents the particle's contribution to the exit measure. At the particle's death two new particles are born and ν is passed to the newborns by fragmentation into two bits.

Joint work with Tom Salisbury.

ALAIN SZNITMAN, ETH Zurich
Random walks and random interlacements

Random interlacements are related to the description of the microscopic texture left in the bulk by random walk on a large discrete torus or random walk on a discrete cylinder with a large base. We discuss in this talk some properties of the model in particular features pertaining to its percolative behavior.