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Estimates for the quenching time of a parabolic equation modeling electrostatic MEMS

We analyze the nonlinear parabolic problem $u_t = \Delta u - \frac{\lambda f(x)}{(1+u)^2}$ on a bounded domain Ω of \mathbb{R}^N with Dirichlet boundary conditions. This equation models a simple electrostatic Micro-Electromechanical System (MEMS) consisting of a thin dielectric elastic membrane with boundary supported at 0 above a rigid ground plate located at -1. When a voltage—represented here by λ —is applied, the membrane deflects towards the ground plate and a snap-through may occur at finite "quenching time" $T_{\lambda}(\Omega, f)$ when it exceeds a certain critical value $\lambda^*(\Omega, f)$ (pull-in voltage). This creates a so-called "pull-in instability" which greatly affects the design of many devices. The challenge is to estimate $\lambda^*(\Omega, f)$ and $T_{\lambda}(\Omega, f)$ in terms of the shape (geometry) and the material properties of the membrane, which can be fabricated with a spatially varying dielectric permittivity profile f. This is a joint work with Yujin Guo.