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On the supra-convergence phenomenon of the cell-centered finite volume method

The contribution investigates the supra-convergence phenomenon which occurs in finite volume methods used to approximate hyperbolic problems on a bounded domain. These methods which take into account the direction where the information comes from are well-adapted for the discretization of such problems. However, even for simple model problems, the theoretical analysis of the error estimate is still a challenging task. One of the main difficulties holds in the fact that the non-uniformity of unstructured meshes brings up an apparent loss of consistency, at least in the finite differences sense. This loss due to the upwind part of the scheme is an artifact of standard convergence proof based on the Lax–Richtmyer theorem. Actually, the scheme maintains the accuracy, the global error behaves in better way than the local error does and converges to zero with the parameter of discretization. This property of enhancement of the truncation error is called supra-convergence.

In order to tackle this lack of consistency, we proceed, for the mathematical analysis, by correcting the error with a geometric corrector introduced for the linear convection problem with constant velocity vector. We first describe the principle of this mathematical analysis. Then we discuss an extension of the notion of geometric corrector to the non-constant velocity case in one dimension since, with the difference of dimension two, an explicit formula of the geometric corrector is available. For a nonlinear conservation law, we are also able to adapt the formula and we can prove that, as long time as the solution remains smooth, the scheme is first order accurate.