FRÉDÉRIC LAGOUTIÈRE, Université Paris-Diderot, 175 rue du Chevaleret, 75013 Paris, France
Analysis of the upwind scheme with probabilities

We provide a probabilistic analysis of the upwind scheme for d-dimensional transport equations on general meshes. One of the purposes of this analysis is to furnish a new "simple" proof of the $1/2$ convergence order of the upwind scheme for non-smooth initial data. The analysis relies on a new interpretation of the scheme, as the *expectation of a random scheme*. We prove that the numerical solution is the expectation of the initial data on the foot of a random characteristic (instead of the initial data on the foot of the exact characteristic of the transport problem). Then the general idea of the analysis is to prove

- first, that the random characteristics are driven *in mean* by the exact ones,
- second, that the fluctuations of the random characteristics around these exact characteristics are of order $Ch^{1/2}$ where h is the maximal cell diameter in the mesh and C only depends on the initial datum and the time: this means that the random characteristics are of *diffusive* type.

This is done *via* Central Limit type Theorems, or, more precisely, with martingale estimates.

We finally prove the $1/2$ order in $L^\infty\big([0,T], L^1(\mathbb{R}^d)\big)$ for BV initial data, and the $1/2-\varepsilon$ rate in $L^\infty\big([0,T], L^\infty(\mathbb{R}^d)\big)$ for Lipschitz-continuous initial data (for any $\varepsilon > 0$).

Besides, this analysis provides a new explanation of the well-known *dissipative* behavior of the upwind scheme, by means of stochastic processes (in the same way as the Brownian motion for the heat equation).