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Stability of high order transport schemes in L^1 and L^∞

Computing the solution of the transport equation

$$\frac{u_j^{n+1}-u_j^n}{\Delta t}+a\frac{u_{j+\frac{1}{2}}^n-u_{j-\frac{1}{2}}^n}{\Delta x}=0.$$

is a fundamental tool in the numerical solution of many hyperbolic problems. We are interested in the numerical analysis of some very high order Finite Volumes explicit schemes recently discussed in the literature.

We shall explain why *all odd order linear explicit schemes derived form the upwind scheme are asymptotically stable in L^1 and L^∞*, that is

$$\|u^n\|_\infty \leq K\|u^0\|_\infty \quad \text{and} \quad \|u^n\|_1 \leq K\|u^0\|_1, \qquad \forall n.$$

This result is a way to bypass the standard obstruction result of Godunov about the nonexistence of high order linear schemes with the maximum principle.

We shall discuss some consequences on cartesian grids of this result for hydrodynamic problems and for the 3D wave equation with nonconstant coefficients. Moreover all the schemes we consider are stable with CFL=1 or 2.