Geometric and Nonlinear Analysis
Analyse géométrique et nonlinéaire
(Org: **Pengfei Guan** (McGill) and/et **Emmanuel Hebey** (Cergy))

VESTISLAV APOSTOLOV, UQAM
Existence and non-existence of extremal Kaehler metrics on ruled manifolds

In this talk I will state (and explain parts of the proof) of a criterion for the existence of an extremal Kaehler metric in certain Kaehler classes on projective line bundles over a polarized manifold of constant scalar curvature. In particular, this allows one to solve the problem of existence of extremal Kaehler metrics on geometrically ruled complex surfaces.

OLIVIER DRUET, PIMS, UBC, Vancouver
Stability of elliptic systems of PDEs

I will discuss the question of stability (and instability) of systems of elliptic PDEs on compact Riemannian manifolds of the form

$$\Delta_g U + AU = |U|^{2^\star - 2} U$$

where $2^\star = \frac{2n}{n-2}$, $U \colon M \mapsto \mathbb{R}^p$ and A is a symmetric $p \times p$ matrix under small perturbations of this interaction matrix A. The aim is to recover the effects of the conformal almost-invariance of the lines of the system (which already appear in the study of the stability of one single equation) and to discover the effects of the interaction matrix A.

AILANA FRASER, University of British Columbia, Department of Mathematics, Vancouver, BC V6T 1Z2
Stable minimal surfaces

It is well known that a complex submanifold of a Kähler manifold must minimize volume in its homology class. Conversely, when is an even dimensional volume minimizing submanifold of a Kähler manifold holomorphic? In this talk I will survey some of what is known about when stable minimal surfaces are holomorphic.

COLIN GUILLARMOU, UNSA Nice, Parc Valrose, 06108 Nice, France
Conformal harmonics, Branson–Gover operators and harmonic forms on Poincaré-Einstein manifolds

Tom Branson and Rod Gover constructed new conformally invariant differential operators acting on k-forms on a conformal compact manifold $(M, [h])$, and a generalization Q_k of Branson Q-curvature for k-forms. The kernel of some of these operators is what they call conformal harmonics.

We show how they are related to harmonic forms on Poincaré–Einstein manifolds with $(M, [h])$ as conformal infinity. In particular, conformal harmonics can be identified with harmonic forms on the bulk with a strong regularity at the boundary, spanning a finite dimensional set.

Joint work with E. Aubry.

NIKY KAMRAN, McGill University, Montreal, Canada
The Penrose process and the wave equation in Kerr geometry

In 1969, Roger Penrose proposed a geometric mechanism for extracting energy and angular momentum from a rotating black hole by exploiting the presence of an ergosphere, that is a region outside the event horizon in which the conserved energy along time-like geodesics can be negative. Soon after the publication of Penrose's paper, Christodoulou established an upper bound for the amount of energy which can be extracted by this process. The wave analogue of the Penrose process, which is called super-radiance, was then investigated by Zel'dovich and Starobinsky for separable solutions of the wave equation corresponding to individual modes.

We will review the Penrose process and super-radiance from the perspective of the Cauchy problem for the scalar wave equation in Kerr geometry, and show that super-radiance can be put into a rigorous mathematical framework. We will quantify the maximal energy gain and show that Christodoulou's bound also holds for scalar waves.

This is joint work with Felix Finster, Joel Smoller and Shing-Tung Yau.

SPIRO KARIGIANNIS, Mathematical Institute, University of Oxford
Conical singularities in G_2 manifolds

I will survey some recent work on compact G_2 manifolds with isolated conical singularities. Specifically I will discuss glueing methods for their desingularization, which involves topological obstructions, as well as the deformation theory (moduli spaces) of such manifolds.

I will also briefly discuss a new construction with Dominic Joyce which could provide the first examples of such manifolds, as well as a possible new construction of smooth compact G_2 manifolds.

OLIVIER LEY, Université François Rabelais, Tours, France
Some uniqueness results for the motion by mean curvature of entire graphs

In 1991, Ecker and Huisken proved that, for any locally Lipschitz continuous initial data $u_0\colon \mathbb{R}^n \to \mathbb{R}$, there exists a smooth solution (for $t > 0$) to the mean curvature equation for graphs

$$\frac{\partial u}{\partial t} - \Delta u + \frac{\langle D^2 u Du, Du\rangle}{1+|Du|^2} = 0 \quad \text{in } \mathbb{R}^n \times (0,\infty),$$
$$u(x,0) = u_0(x) \quad \text{in } \mathbb{R}^n.$$

We are concerned with the issue of uniqueness. In the existence result, the intriguing point is that no assumption is made on the growth of u_0 at infinity and therefore the solution u itself can have an arbitrary growth. We use different approaches including geometrical and analytical tools to provide uniqueness results in the following cases:

(i) when u_0 is convex (or more generally "convex at infinity"),

(ii) when $n = 1$,

(iii) when u_0 is radial, or

(iv) when assuming some polynomial growth conditions.

Joint works with G. Barles, S. Biton, M. Bourgoing, P. Cardaliaguet and E. Chasseigne.

JUNFANG LI, CRM & McGill University
The quermassintegral inequalities for starshaped domains

We give a simple proof of the isoperimetric inequality for quermassintegrals of non-convex starshaped domains, using a result of Gerhardt and Urbas on an expanding geometric curvature flow.

FREDERIC ROBERT, Université de Nice–Sophia Antipolis
24. Positivity issues of biharmonic Green's functions under Dirichlet boundary conditions

In general, higher order elliptic equations and boundary value problems like the biharmonic equation or the linear clamped plate boundary value problem do not enjoy neither a maximum principle nor a comparison principle or—equivalently—a positivity preserving property. It is shown that, on the other hand, for bounded smooth domains $\Omega \subset \mathbb{R}^n$, the negative part of the corresponding Green's function is "small" when compared with its singular positive part, provided that $n \geq 3$.

JEAN-MARC SCHLENKER, Université Toulouse 3
On the renormalized volume of quasifuchsian manifolds

The renormalized volume of quasifuchsian hyperbolic 3-manifolds was originally introduced for physical reasons. Takhtajan and Zograf (and others) discovered that it provides a Kähler potential for the Weil–Petersson metric on Teichmüller space. We will give an elementary, differential-geometric account of this result. It can be extended to quasifuchsian manifolds having cone singularities along infinite lines, yielding results on the Teichmüller space of hyperbolic metrics with cone singularities (of prescribed angles) on a closed surface.

Joint works with K. Krasnov, C. Lecuire, S. Moroianu.

ALINA STANCU, Concordia University, Department of Mathematics and Statistics
On the volume product functional

Given a convex body in $\mathbb{R}^{n+1}$ containing the origin, the volume product functional associates to it the value $\mathrm{Vol}(K) \cdot \mathrm{Vol}(K^*)$, where K^* denotes the dual polar body of K with respect to the origin. This number is the object of Santaló's inequality as well as that of Mahler's conjecture. We will present some results related to bounds of this functional.

McKENZIE WANG, McMaster University
Examples of Ricci Solitons

Ricci solitons are special solutions of the Ricci flow given by a 1-parameter family of diffeomorphisms and homotheties. They may be viewed as generalizations of Einstein metrics. They also occur when analysing singularities of the Ricci flow or when one tries to derive matrix Harnack inequalities.

I will describe some new examples of non-Kahler Ricci solitons which generalise those constructed by T. Ivey.

JIE XIAO, Memorial University, St. John's, NL A1C 5S7
Some Geometric Criteria for L^2 Sobolev/Nash/Log-Sobolev Inequalities on Complete Noncompact Riemannian Manifolds

In this talk we will discuss certain generic and optimal geometric criteria for the mutually interacted Sobolev, Nash, logarithmic Sobolev inequalities of L^2 type on $3 \leq n$-dimensional, complete, non-compact, Riemannian manifolds. More attention is paid on those aspects connecting the volumes and the Green's functions of regular bounded open subsets of the manifolds.