
Singular perturbations and the Ginzburg-Landau model (CAIMS)
Perturbations singulières et le modèle de Ginzburg-Landau (SCMAI)
(Org: **Stan Alama** and/et **Lia Bronsard** (McMaster))

YANIV ALMOG, Department of Mathematics, LSU, Baton Rouge, LA 70803

Radially symmetric local minimizers of the Ginzburg–Landau energy functional

We discuss degree one radially symmetric solutions to the Ginzburg–Landau equations in the presence of magnetic fields. We first discuss their existence (and non-existence) in the whole plane, where they are always unstable. Then we discuss their stability on a finite disc.

PATRICIA BAUMAN, Purdue University, West Lafayette, IN, USA

Stable Solutions of a Variational Model for Bentcore Liquid Crystals

We present a nonlinear variational model for bent-core (banana-shaped) liquid crystals which includes energy terms including polarization, smectic and chiral effects, elasticity, and surface tension. We analyze stable solutions in a physically realistic regime involving a free boundary problem for the energy and the existence of a limiting-energy minimizer.

DMITRY GOLOVATY, The University of Akron, Akron, OH 44325, USA

A Ginzburg–Landau-type model of carbon nanotubes

We demonstrate that a multiwalled carbon nanotubes—an interacting system of concentric carbon layers—can be effectively described by a Ginzburg–Landau-type energy with a potential that has multiple wells. The number of wells depends on the geometric characteristics of the tube and leads to the experimentally observed polygonization effect.

RADU IGNAT, Département de Mathématiques, Université Paris-Sud 11, Bât. 425, 91405 Orsay, France

A Γ -convergence result for Néel walls in micromagnetics

The Néel wall is a dominant transition layer in thin ferromagnetic films. It is characterized by a one-dimensional in-plane rotation connecting two (opposite) directions of the magnetization. It is a two length scale object: a small core with fast varying rotation and two logarithmically decaying tails. There are three confining mechanisms for the Néel tails: the anisotropy of the material, steric interaction with the sample edges and steric interaction with the tails of neighboring Néel walls. We describe these models that correspond to three non-convex and non-local variational problems depending on a small parameter. The aim of the talk is to prove the Γ -convergence of these three energies. The limiting magnetizations have a finite number of jumps corresponding to the same angle wall. The Γ -limit counts the number of walls and the energy of a wall is quadratically in the angle wall.

VINCENT MILLOT, University of Cergy-Pontoise

Symmetry of locally minimizing solutions for the 3D Ginzburg–Landau system

In this talk, we will classify nonconstant entire local minimizers of the standard Ginzburg–Landau functional for maps in $H_{loc}^1(\mathbb{R}^3; \mathbb{R}^3)$ satisfying a natural energy bound. Up to isometries, such solution of the Ginzburg–Landau system is given by an explicit solution equivariant under the action of the orthogonal group.

This is a joint work with Adriano Pisante.

DAN PHILLIPS, Purdue University, West Lafayette, IN, US

Smectic energies and existence theorems for liquid crystals

Smectic C liquid crystals form layered structures with a well defined layer thickness, where the layers are made up of rod like liquid crystal molecules, each of which tends to tilt from the layer normal by an angle θ . Liquid crystal configurations are described by a pair (ψ, \mathbf{n}) . The director field \mathbf{n} is a unit vector field representing the distribution of the molecules' long axes. ψ is a complex order parameter giving the layer structure. We investigate a 2nd order free energy, and prove the existence of minimizing pairs representing stable configurations.

ITAI SHAFRIR, Technion-I.I.T., 32000 Haifa, Israel

A nonlocal problem arising in the study of magneto-elastic interactions

The energy of magneto-elastic materials is described by a nonconvex functional. Three terms of the total free energy are taken into account: the exchange energy, the elastic energy and the magneto-elastic energy, usually adopted for cubic crystals. We focus our attention on a simplified one dimensional problem with penalty and study the stationary problem and the gradient flow of the associated Ginzburg–Landau type functional. We prove existence and uniqueness of a classical solution which tends asymptotically (for subsequences) to a stationary point of the energy functional.

This is a joint work with M. Chipot, V. Valente and G. Vergara Caffarelli.

PETER STERNBERG, Indiana University

Time-dependent Ginzburg–Landau model with an applied current

We study formally and rigorously the bifurcation to steady and time-periodic states in a model for a thin superconducting wire in the presence of an imposed current. Exploiting the PT-symmetry of the equations at both the linearized and nonlinear levels, and taking advantage of the collision of real eigenvalues leading to complex spectrum, we obtain explicit asymptotic formulas for the stationary solutions, for the amplitude and period of the bifurcating periodic solutions and for the location of their zeros or “phase slip centers” as they are known in the physics literature. In so doing, we construct a center manifold for the flow and give a complete description of the associated finite-dimensional dynamics.

This is joint work with Qingfeng Ma, Jacob Rubinstein and Kevin Zumbrun.