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Casimir Cascades in Two-Dimensional Turbulence

The Kraichnan–Leith–Batchelor theory of two-dimensional turbulence is based on the fact that the nonlinear terms of the two-dimensional Navier–Stokes equation conserve both energy and enstrophy. In an infinite domain and in the limit of infinite Reynolds number, the net energy and enstrophy transfers out of a low-wavenumber forcing region must consequently be independent of wavenumber. The resulting dual cascade of energy to larger scales and enstrophy to smaller scales is readily observed in numerical simulations of two-dimensional turbulence in a finite domain.

While it is well known that the nonlinearity also conserves the global integral of any arbitrary C^1 function of the scalar vorticity field, the direction of transfer of these quantities in wavenumber space remains unclear. Numerical investigations of this problem are hampered by the fact that pseudospectral simulations, which necessarily truncate the wavenumber domain, do not conserve these higher-order Casimir invariants. In this work we develop estimates for the degree of nonconservation of the Casimir invariants and demonstrate using sufficiently well-resolved simulations that the fourth power of the vorticity cascades to small scales.

JAHANSHAH DAVOUDI, University of Toronto

Variability of the equilibrium mass flux distribution with altitude

The equilibrium fluctuations of a field of cumulus clouds under homogeneous large-scale forcing was recently shown to satisfy the Gibbs canonical ensemble from statistical mechanics. In the limit of noninteracting convective cells an analytical expression for the distribution function of total mass flux over a region of given size was derived.

We examine the consistency of the Gibbs canonical ensemble with the mass flux fluctuations in higher altitudes by means of cloud resolving simulations in radiative convective equilibrium. The obtained data exhibits a systematic height dependence in the mass flux probability distribution.

YOHANN DUGUET, Department of Mechanical Engineering, KTH, Stockholm, Sweden

The role of finite-amplitude solutions in transition to turbulence in pipe flow

Transition to turbulence in a circular pipe is an old puzzling problem for which a dynamical system description appears most enlightning. We will explain the recent notion of ‘laminar/turbulent boundary’ in phase-space and what it teaches us about transitional dynamics. Numerical investigation shows that this subspace of phase-space appears to be organised around a complex network of repelling finite-amplitude solutions, among which three-dimensional travelling waves also and relative periodic orbits. We will detail these ideas on pipes of short length and then extend it to more realistic longer pipes, allowing for spatially localised structures (‘puffs’).

RICHARD KARSTEN, Acadia University
Assessment of Tidal Current Energy in Minas Passage, Bay of Fundy

The Bay of Fundy has the world's highest tides. In particular, the Minas Basin has tides with a range of over 12m. The Minas Passage, which connects Minas Basin to the Bay of Fundy, has mean tidal currents of over 3m/s making it a promising location for tidal turbines. In this talk we examine the potential power that could be extracted from Minas Passage and the effect that extracting the power would have on the surrounding tides. A mathematical model is used to predict the effect of turbine drag on the flow through the Minas Passage and the tidal amplitude in the Minas Basin. The theory is compared to two-dimensional, finite-element numerical simulations of the Bay of Fundy-Gulf of Maine system. Together, they suggest that a maximum of 7 GW of power can be extracted by turbines. The simulations also show that any power extraction in Minas Passage pushes the Gulf of Maine-Bay of Fundy system closer to resonance with the forcing tides resulting in increased tidal amplitudes throughout the Gulf of Maine. While extraction of the maximum power will result in significant changes, over 2.5 GW of power can be extracted with less than a 5% change in the tidal amplitude at any location. Finally, we examine how isolated turbines and turbine fences might be best located in the Minas Passage by examining the fluid dynamics of flow past a turbine.

NICHOLAS KEVLAHAN, McMaster University, Hamilton, ON L8S 4K1
Three-dimensional Floquet stability analysis of the wake in cylinder arrays

Three-dimensional stability of the periodic wake of tightly packed rotated and inline cylinder arrays is investigated for $60 \leq Re \leq 270$. Results are compared with existing numerical and experimental studies for an isolated cylinder. Numerical Floquet analysis shows that the two-dimensional wakes of the rotated and inline arrays with spacing $P/D = 1.5$ become unstable at $Re_c = 64 \pm 0.5$ and $Re_c = 132 \pm 1$ respectively. Two-dimensional vortex shedding flow is unlikely in practice for such flows. The dominant spanwise wavelength is $\lambda/D = 0.9 \pm 0.1$ for the rotated array at $Re = 100$ and $\lambda/D = 3.0 \pm 0.1$ for the inline array at $Re = 200$. Three-dimensional simulations show excellent agreement with the Floquet analysis for the rotated case, and reasonable agreement for the inline case. The instability mechanism appears to be similar to Mode A for an isolated cylinder, although the structure of the three-dimensional vorticity is different due to the spatial periodicity of the flow. Unlike the isolated cylinder, both array flows are unstable as $\lambda \to \infty$ (like a thin shear layer). This is the first investigation of three-dimensional wake instability in cylinder arrays, a problem of significant practical and theoretical interest.

SERPIL KOCABIYIK, Memorial University of Newfoundland
Free surface flow with an oscillating cylinder based on a two-fluid model

An accurate computational method for a viscous incompressible fluid flow past a circular cylinder beneath the free surface is presented. The method is designed for studying free surface problems with arbitrary moving circular cylinders. The method of solution is based on a finite volume discretization of the two-dimensional continuity and unsteady Navier–Stokes equations in their pressure-velocity formulation on a fixed Cartesian grid. Well-posed boundary conditions are enforced at the inflow and outflow boundaries since they ensure correct physical development of the flow near the computational domain boundaries. The free surface boundary conditions are applied implicitly by using a two-phase flow technique which treats the air phase and the fluid phase as a single fluid with variable material properties. The displacement of a free surface is tracked by using the volume of fluid method. The fractional area/volume obstacle representation method is combined with the cut cell method to improve the accuracy of the spatial discretization of a fluid-body interface. The numerical algorithm is verified by applying it to the special case of uniform flow past a cylinder undergoing forced oscillations in streamwise direction in the presence of a free surface.

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ROUSLAN KRECHETNIKOV, University of Alberta, Edmonton, Canada
On the transition to turbulence problem

In this talk we systematically discuss the key points advocated in the literature by various schools of thought and relate them to the known rigorous facts about the Navier–Stokes equations and to the well-established rigorous mathematical frameworks. Besides providing a coherent story from the analysis point of view, we also discuss two main pillars upon which the current discussion in the literature is based, namely the crucial effect of non-normality of the linear operators and the energy-conserving nature of the nonlinear terms. Finally, we propose alternative ways to make a progress on this long-standing problem.

GREGORY LEWIS, University of Ontario Institute of Technology (UOIT)
Transitions in an air-filled differentially heated rotating annulus

We discuss the transition from axisymmetric to wave solutions in a mathematical model of an air-filled differentially heated rotating annulus. We use the Navier–Stokes equations in the Boussinesq approximation to model the flow of the air. An investigation of the double Hopf bifurcations that occur in the model indicate the existence of stable mixed azimuthal mode flows. The results for this near unity Prandtl number fluid are compared to those for larger Prandtl number fluids.

SHERWIN MASLOWE, McGill, Montreal, Quebec
Stability of Swirling Flows

A swirling flow is one having both azimuthal and axial velocity components, $V(r)$ and $W(r)$, say, with radial velocity $U = 0$. The stability of such flows is pertinent to many applications in engineering and meteorology such as tornadoes and the trailing vortices behind aircraft. In this talk, the theorems governing the linear stability of such flows will be briefly reviewed and some new numerical results will be presented. An analysis describing the propagation of vortex Kelvin waves with nonlinear critical layers will then be outlined and a solution given that is valid in the large Reynolds number limit.

FRANCIS POULIN, University of Waterloo
The effect of stochasticity on linear waves

We study the propagation of linear waves through a medium whose properties can vary both in space and in time. If we assume that the wave speed is separable then we obtain that the stability of the waves is determined by a generalized Mathieu's equation (Poulin & Flierl, 2008). By numerically integrating this equation using a symplectic method we determine that the stochasticity can either diffuse or localize the wave depending on the nature of the medium.

BARTOSZ PROTAS, McMaster University, Hamilton, Ontario, Canada L8S 4K1
Shape Optimization and Free Boundary Problems in Fluid Mechanics

In this presentation we discuss a technique for the computational solution of some classes of free boundary problems arising in fluid mechanics. In such problems the shape of the domain where the governing PDEs are stated in unknown and must be determined as a part of the solution of the problem. As examples we consider the following two model problems:

(i) the *direct* Stefan problem where one seeks to find the phase-change interface between the liquid and solid phases in the presence of a contact line, and

(ii) the *inverse* problem of Vortex Design where one seeks to determine the boundary conditions for the incompressible Euler equations, so that the vortex region in the flow will have a prescribed shape.

We show that both of these problems can be efficiently solved using methods of the PDE-constrained optimization, where the unknown shape of the domain boundary is sought as a minimizer of a suitable cost functional. We demonstrate how the optimality conditions for such problems are derived using the *shape calculus* which allows one to differentiate solutions of PDEs with respect to the geometry of the domain. Such optimal domain shapes can be found computationally using a gradient-based algorithm, where the gradient of the cost functional can be conveniently computed by solving an adjoint PDE system. In addition to discussing the mathematical foundations of this method we will also present computational examples illustrating its performance on the Stefan and Vortex Design problems.

Joint work with Oleg Volkov.

MAREK STASTNA, University of Waterloo
Numerical modeling of internal wave-boundary layer interaction

Internal waves in stratified fluids are a ubiquitous feature of both the oceans and the atmosphere. Many classical theories naturally assume that the fluid in which the wave propagate is inviscid. Over the past few years there has been a gradual accumulation of experimental and numerical work that has exposed the rich interplay between internal waves and the viscous boundary layer. I will discuss examples of internal solitary-like waves propagating over an undulating bottom, generating vortices as they go. Using Lagrangian particle tracking, I will demonstrate that the vortices transport near bottom fluid up into the water column in a coherent manner. Using suites of simulations I will demonstrate that there is an optimal wavelength of topography undulations that leads to a maximum in kinetic energy production due to vortex production. When the thermo or pycnocline lies near the bottom, the vortices deform the pycnocline in a significant way and can potentially serve to partially destroy the pycnocline, thereby altering the waveguide for waves propagating behind the leading wave. Time permitting I will compare the results for vertically-trapped waves with those found for viscous critical reflection of vertically propagating internal waves.

LENNAERT VAN VEEN, Concordia University, 1455 de Maisonneuve Blvd West, Montreal, QC, Canada H3G 1M8
The computation of 2D unstable manifolds in models of turbulent shear flow

The publication of a landmark paper by Kawahara and Kida (2001) on the relevance of unstable periodic orbits to the dynamics of shear flow has initiated intense efforts to explain such phenomena as bursting and subcritical transition in terms of dynamical systems theory. Results similar to and stronger then those of Kawahara and Kida were since found in a variety of geometries, such as flow through pipes, ducts and channels. In all these geometries there exists a laminar flow profile which remains asymptotically stable up to high, sometimes infinite, Reynolds number. Thus, the onset of turbulence cannot be explained in terms of a straightforward bifurcation scenario as is often found in rotating or differentially heated flows. Instead, the relevant dynamical structures seem to be periodic orbits which live on the boundary of the domain of attraction of the laminar flow profile in phase space.

We can study the geometry of the basin boundary through the stable and unstable manifolds of such orbits. However, the computation of manifolds embedded in a high-dimensional phase space is a hard task. In this presentation I will show some recent results obtained with a low-order model of shear flow and, if time allows, with a full-fledged simulation of plane Couette flow.

DIVAKAR VISWANATH, University of Michigan
The critical layer and transition to turbulence in shear flows

Roughly speaking, lower branch solutions are traveling wave or equilibrium solutions of shear flows that are in-between laminar flow and turbulence. We describe the computation of a lower-branch family of traveling waves in pipe flow up to $\mathrm{Re} = 75000$ (Re = Reynolds number). At high Re, these traveling waves develop a critical layer away from the pipe wall. Such critical layers could be useful to visualize the structure of puffs observed in transitional flow. Because of the small number of unstable eigenvalues, the lower-branch solutions also appear to be connected to transition to turbulence as we will show.

MICHAEL WAITE, University of Victoria, PO Box 3060 Stn CSC, Victoria, BC V8W 3R4
Atmospheric boundary layer effects in a two-mode multicloud model

Intermediate models with low vertical resolution are an important tool in the study of tropical convection and convectively coupled waves. They are comprehensive enough to reproduce many features of convection, yet tractable enough to permit detailed analysis, particularly of wave structure and stability. In this talk, we employ such a model to examine the dynamics of the atmospheric boundary layer in convectively-coupled gravity waves. Bulk boundary layer equations, which include the effects of environmental and convective mass fluxes, are developed using ideas from Stevens (Theor. Comput. Fluid Dyn. **20**(2006), 279). These equations are coupled to the Khouider–Majda multicloud model, a system of shallow-water equations for two baroclinic modes with parameterized deep, congestus, and stratiform convection (Khouider & Majda, J. Atmos. Sci. **63**(2006), 1308). Linear stability analyses and preliminary nonlinear simulations will be discussed.

THOMAS WIHLER, McGill University, 805 Sherbrooke W., Montreal, QC, Canada H3A 2K6
hp-Discontinuous Galerkin methods for the Stokes equations

We will discuss hp-discontinuous Galerkin finite element discretizations for the Stokes equations in two space dimensions. In hp-approaches both the element sizes as well as the approximation orders can vary locally. In this way, hp-methods can adjust to the local solution behavior very efficiently.

We shall first present some a priori results for smooth solutions with local singularities in polygonal domains. Specifically, we will show how exponential rates of convergence can be obtained in the numerical approximations.

Secondly, we will present an hp-error indicator (based on a suitable hp-a posteriori error analysis) that can be combined with an hp-adaptive algorithm resulting in automatic refinements of the approximation spaces. In addition, some numerical experiments shall be given.