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Casimir Cascades in Two-Dimensional Turbulence

The Kraichnan–Leith–Batchelor theory of two-dimensional turbulence is based on the fact that the nonlinear terms of the two-dimensional Navier–Stokes equation conserve both energy and enstrophy. In an infinite domain and in the limit of infinite Reynolds number, the net energy and enstrophy transfers out of a low-wavenumber forcing region must consequently be independent of wavenumber. The resulting dual cascade of energy to larger scales and enstrophy to smaller scales is readily observed in numerical simulations of two-dimensional turbulence in a finite domain.

While it is well known that the nonlinearity also conserves the global integral of any arbitrary C^1 function of the scalar vorticity field, the direction of transfer of these quantities in wavenumber space remains unclear. Numerical investigations of this problem are hampered by the fact that pseudospectral simulations, which necessarily truncate the wavenumber domain, do not conserve these higher-order Casimir invariants. In this work we develop estimates for the degree of nonconservation of the Casimir invariants and demonstrate using sufficiently well-resolved simulations that the fourth power of the vorticity cascades to small scales.