RYAN BUDNEY, Mathematics and Statistics, University of Victoria, Victoria, BC, Canada V8W 3P4

The space of embeddings of S^1 in S^3 and related topics

I will provide a status-update on what is known about the homotopy-type of the space of embeddings of the circle in the 3-sphere, $\mathrm{Emb}(S^1, S^3)$. $\mathrm{Emb}(S^1, S^3)$ fibers over a Stiefel manifold with fiber the space of "long knots" $\mathrm{Emb}(R, R^3)$, these are the smooth embeddings of R in R^3 which agree with a fixed linear inclusion $R \to R^3$ outside of a ball.

By the work of Allen Hatcher, the components of $\mathrm{Emb}(R, R^3)$ are $K(\pi, 1)$ spaces (components are precisely isotopy-classes of knots). I worked out a procedure to compute the fundamental groups of these components, but the answer requires

(1) knowledge of the JSJ-decomposition of the knot complement,

(2) knowledge of the geometric structures on the complement split along its JSJ-decomposition, and

(3) knowledge of a certain signed-symmetric representation of the isometry groups of certain "almost Brunnian" hyperbolic link complements (these are the hyperbolic link complements that arise in the JSJ-decompositions of knots).

My overriding motivation is that I believe it is possible to have a "closed form" description of the homotopy type of the spaces $\mathrm{Emb}(S^1, S^3)$ and $\mathrm{Emb}(R, R^3)$, where "closed form" in this context means "$\mathrm{Emb}(R, R^3)$ has the homotopy type of a collection of connected spaces X, where X is generated from the 1-point space via 3 simple bundle operations, where the base spaces are S^1, $S^1 \times S^1$, or coloured configuration spaces in R^2, and the fibers are products of previous spaces in the collection X, and the monodromy is given explicitly from an elementary table of monodromies." In principle this should give a description of $H_*\big(\mathrm{Emb}(R, R^3)\big)$ as a certain "mangled" bar construction, for lack of a better word.