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Deformation theory of algebraic structures

The purpose of this talk is to define and study the deformation theory of universal algebras based on the recent use of higher algebraic objects, operads and props, that model these categories.

Many categories of algebras like associative algebras, Lie algebras, Gerstenhaber algebras, BV-algebras, Lie bialgebras or associative bialgebras, for instance, are governed by one algebraic object called an operad or more generally a prop. Any construction performed on the level of operads or props give universal results to any algebras encoded by this operad or prop. We will define the deformation theory of morphisms of prop/operads à la Quillen. For instace, we recover Hochschild (co)homology for associative algebras, Chevalley–Eilenberg (co)homology for Lie algebras or Gerstenhaber–Shack (co)homology for associative bialgebras.

Using operadic homological algebra (twisting morphisms, bar and cobar constructions), we will make this cohomology theories explicit. This will allow to prove that these deformation theories are always governed by a homotopy Lie algebra, thus verifying Deligne's statement that "any deformation theory should be governed by a differential graded Lie algebra". For instance, this shows that Gerstenhaber–Shack bicomplex is endowed with a natural structure of homotopy Lie algebra.

Any realization of these chain complexes is based on a cofibrant resolution of the associated prop. Koszul duality theory gives an efficient method to make explicit such resolutions. Unfortunately, many props fail to be Koszul, so we will present other methods to get cofibrant resolutions of props.