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Purely periodic beta-expansions

It is well known that real numbers with a purely periodic decimal expansion are rationals having, when reduced, a denominator coprime with 10. We extend here this result to beta-expansions with a Pisot base beta which is not necessarily a unit. Beta-numeration generalises usual binary and decimal numeration: taking any real number $\beta > 1$, it consists in expanding numbers $x \in [0, 1]$ as power series in base β^{-1} with digits in $\mathcal{D} = \{0, \ldots, \lceil\beta\rceil - 1\}$. We characterize real numbers having a purely periodic expansion in such a base in terms of an explicit set, called a central tile, which is shown to be a graph-directed self-affine compact subset of non-zero measure which belongs to the direct product of Euclidean and p-adic spaces. We focus furthermore on the gamma function $\gamma(\beta)$ defined as the supremum of the set of elements v in $[0, 1]$ such that every positive rational number p/q, with $p/q \leq v$ and q coprime with the norm of β, has a purely periodic β-expansion. We will also survey the connections between geometric representations of beta-shifts as central tiles, and finite-to-one covers of hyperbolic toral automorphisms, such as discussed, *e.g.*, by Vershik, Sidorov or Schmidt by considering the group of fundamental homoclinic points in the Pisot case.

This work is a joint work with S. Akiyama, G. Barat and A. Siegel.