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Quartic spline collocation for fourth-order boundary value problems on rectangles with an application to the biharmonic Dirichlet problem

Bi-quartic spline collocation methods for the numerical solution of fourth-order boundary value problems on rectangular domains are presented. A particular instance of these methods is applied to the biharmonic Dirichlet problem.

The bi-quartic spline collocation methods use the midpoints of a uniform partition, the boundary midpoints and the corners as collocation points. While the standard bi-quartic spline method provides second-order approximations, two bi-quartic spline collocation methods, the one-step (extrapolated) and the three-step (deferred-correction) methods, produce approximations which are sixth order at gridpoints and midpoints, and fifth order at other points. Both are based on high order perturbations of the differential and boundary conditions operators.

The properties of the three-step method matrices arising from a restricted class of problems are studied. Analytic formulae for the eigenvalues and eigenvectors are developed, and related to those arising from quadratic-spline collocation matrices. These properties lead to a fast solver for the biharmonic Dirichlet problem on rectangles. The fast solver is based on Fast Fourier Transforms applied to an auxiliary biharmonic problem with Dirichlet and second derivative boundary conditions along the two opposite boundaries, and on preconditioned GMRES applied to a problem related to the two opposite boundaries. By analyzing the eigenvalues of the preconditioned matrix, the solver is shown to have complexity $O(N^2 \log N)$ on a $N \times N$ partition. Numerical experiments from a variety of problems, including practical applications and problems more general than the analysis assumes, verify the accuracy of the discretization scheme and the effectiveness of the fast solver.

Joint work with Jingrui Zhang.