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Convex and coconvex polynomial approximation in the uniform norm

A survey on the results by K. Kopotun, D. Leviatan, the author, and others.

Let $s \in \mathbb{N}$, $-1 < y_s < \cdots < y_1 < 1$, $Y_s = \{y_i\}_{i=1}^s$, $\Delta^{(2)}(Y_s)$ be the set of continuous on $[-1,1]$ functions, which are convex on $[y_1, 1]$, concave on $[y_2, y_1]$, etc., $\Delta^{(2)}(Y_0)$ be the set of convex continuous on $[-1,1]$ functions, $\|\cdot\|$ be a uniform norm on $[-1,1]$, $\mathbb{P}_n$ be the space of algebraic polynomials of degree less that n, and

$$E_n^{(2)}(f, Y_s) := \inf_{P_n \in \mathbb{P}_n \cap \Delta^{(2)}(Y_s)} \|f - P_n\|$$

be the error of the best uniform coconvex approximation of f.

For $k \in \mathbb{N}$, $r \in \mathbb{N}_0$ and function $f \in C^{(r)} \cap \Delta^{(2)}(Y_s)$ we will discuss the validity of the inequality

$$E_n^{(2)}(f, Y_s) \leq \frac{C}{n^r} \omega_k\Big(\frac{1}{n}, f^{(r)}\Big), \quad n \geq N,$$

where ω_k are moduli of smoothness of different types.