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Infinite Products of Random Matrices and Repeated Interaction Dynamics

Let Ψ_n be a product of n independent, identically distributed random matrices M, with the properties that Ψ_n is bounded in n, and that M has a deterministic (constant) invariant vector. Assuming that the probability of M having only the simple eigenvalue 1 on the unit circle does not vanish, we show that Ψ_n is the sum of a fluctuating and a decaying process. The latter converges to zero almost surely, exponentially fast as $n \to \infty$. The fluctuating part converges in Cesaro mean to a limit that is characterized explicitly by the deterministic invariant vector and the spectral data of $\mathbb{E}[M]$ associated to 1. No additional assumptions are made on the matrices M; they may have complex entries and not be invertible.

We apply our general results to two classes of dynamical systems: inhomogeneous Markov chains with random transition matrices (stochastic matrices), and random repeated interaction quantum systems. In both cases, we prove ergodic theorems for the dynamics, and we obtain the form of the limit states.