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A group representation for the anti-Pappian design

It was shown by H. Schröter [Nachr. Ges. Wiss. Göttingen 1889, 193–236] that among the ten combinatorially possible $(10, 3)$ designs, only one cannot be realized in a projective plane over any field. In view of this, this D10 is known in the literature as an anti-Pappian design. In 1954, R. Lauffer [Math. Nachrichten, vol. 11] gave a representation of this design over the infinite division ring of quaternions. This proves that the design, viewed as a ternary implicational system, is strictly consistent, meaning that it is impossible to formally derive $x = y$ from the ten defining equations. Here we give a group representation of rank 3 for this configuration and show that 3 is the minimal possible rank for such a representation. Apart from demonstrating the combinatorial consistency of the anti-Pappian design, this gives a new proof of the fact that this design cannot be realized in any finite Desarguesian plane. This is part of an unpublished set of notes on group representations written in collaboration with Barry Wolk and the late Professor Nathan Mendelsohn.