JAVAD MASHREGHI, Laval

Integral representations of the derivatives of functions in $\mathcal{H}(b)$

Let $H^p(\mathbb{C}_+)$ stand for the Hardy space of the upper half plane $\mathbb{C}_+$, and for $\varphi \in L^\infty(\mathbb{R})$, let T_φ stand for the Toeplitz operator defined on $H^2(\mathbb{C}_+)$ by

$$T_\varphi(f) := P_+(\varphi f), \quad \big(f \in H^2(\mathbb{C}_+)\big),$$

where P_+ denotes the orthogonal projection of $L^2(\mathbb{R})$ onto $H^2(\mathbb{C}_+)$. Then, for $\varphi \in L^\infty(\mathbb{R})$, $\|\varphi\|_\infty \leq 1$, the de Branges–Rovnyak space $\mathcal{H}(\varphi)$, associated to φ, consists of those $H^2(\mathbb{C}_+)$ functions which are in the range of the operator $(\mathrm{Id} - T_\varphi T_{\overline{\varphi}})^{1/2}$. It is a Hilbert space when equipped with the inner product

$$\langle (\mathrm{Id} - T_\varphi T_{\overline{\varphi}})^{1/2} f, (\mathrm{Id} - T_\varphi T_{\overline{\varphi}})^{1/2} g \rangle_\varphi = \langle f, g \rangle_2,$$

where $f, g \in H^2(\mathbb{C}_+) \ominus \ker(\mathrm{Id} - T_\varphi T_{\overline{\varphi}})^{1/2}$. In particular, if b is an inner function, then $(\mathrm{Id} - T_b T_{\overline{b}})$ is an orthogonal projection and $\mathcal{H}(b)$ is a closed (ordinary) subspace of $H^2(\mathbb{C}_+)$ which coincides with the so-called model spaces $K_b = H^2(\mathbb{C}_+) \ominus b H^2(\mathbb{C}_+)$.

We give some integral representations for the boundary values of derivatives of functions of the de Branges–Rovnyak spaces $\mathcal{H}(b)$, where b is an extreme point of the unit ball of $H^\infty(\mathbb{C}_+)$.