
DAVID MINDA, University of Cincinnati, Cincinnati, Ohio
Geometric variations of Schwarz's Lemma

The classical version of Schwarz's Lemma deals with holomorphic self-maps of the unit disk \mathbb{D} that fix the origin; the extremal functions for Schwarz's Lemma are rotations about the origin. We consider holomorphic maps of \mathbb{D} into a region Ω that satisfies some geometric property that holds for the unit disk. For example, Ω has diameter at most 2. There are regions of diameter 2 that are not contained in a disk of radius 1, so this case properly contains the classical framework. Landau and Toeplitz considered this situation in 1907. Other geometric conditions on Ω involve the area, length of the boundary and higher-order diameters, including the transfinite diameter. In all cases we obtain sharp analogs of the classical Schwarz Lemma and identify the extremal functions.