DANIELA KRAUS, University of Würzburg

Critical points of inner functions, nonlinear partial differential equations, and an extension of Liouville's theorem

Theorem 1 *Let $\{z_j\} \subseteq \mathbb{D}$ be a Blaschke sequence. Then there exists a Blaschke product with critical points $\{z_j\}$ (counted with multiplicity) and no others.*

The case of finitely many critical points has been proved earlier by Heins 1962, Wang & Peng 1979, Bousch 1992 and Zakeri 1996 (topological proofs) and by Stephenson 2005 (discrete methods). It has found applications in complex dynamics by Milnor.

The proof of Theorem 1 is based on an extension of Liouville's classical representation theorem for solutions of the partial differential equation $\Delta u = 4e^{2u}$ combined with methods from nonlinear elliptic PDE. Our work is closely related to the Berger–Nirenberg problem in differential geometry.

Joint work with Oliver Roth.