
Set Theory and Infinitary Combinatorics
Théorie des ensembles et combinatoire infinie
(Org: **Stevo Todorcevic** (Toronto))

ILIJAS FARAH, York University, Toronto, ON

Condensed groups

A topological group is extremely amenable if it does not have a fixed-point free action on a compact Hausdorff space. I'll talk about a new example of extremely amenable groups of functions with topology of convergence with respect to a diffused submeasures. The main result unifies earlier work of Christensen–Herer and Pestov. We deduce our results from a Ramsey-type result and give an example showing there is no concentration of measure in this context.

This is a joint work with Slawek Solecki.

VALENTIN FERENCZI, Université Pierre et Marie Curie – Paris 6, Equipe d'Analyse Fonctionnelle, Boîte 186, 4, place Jussieu, 75252 Paris Cedex 05, France

The class of separable reflexive Banach spaces is strongly bounded

A class C of separable Banach spaces is said to be strongly bounded if whenever A is an analytic subset of C , there exists a space in C which contains an isomorphic copy of any X in A .

It is proved that the classes REFL of separable reflexive spaces and SD of spaces with separable dual are strongly bounded. This gives another proof of a result of E. Odell and T. Schlumprecht answering a question of J. Bourgain: there exists a separable reflexive Banach space which is universal for the class of separable uniformly convex spaces.

Joint work with Pandelis Dodos.

VERA FISCHER, York University, 4700 Keele Street, Toronto, Ontario M3J 1P3

The Consistency of Arbitrarily Large Spread Between the Bounding and the Splitting Numbers

In 1984, S. Shelah obtained the consistency of $\mathfrak{b} = \omega_1 < \mathfrak{s} = \omega_2$ using countable support iteration of proper forcing notions. The method can not be further generalized since subsequent iterations would collapse the continuum. However finite support iteration of c.c.c. forcing notions does not have this disadvantage and we succeed to extend the above result obtaining a model of $\mathfrak{b} = \omega_1 < \mathfrak{s} = \mu$ for μ arbitrary regular cardinal. The c.c.c. forcing notions which we use are closely related to the partial order used originally by S. Shelah.

PIOTR KOSZMIDER, Universidade de São Paulo

Combinatorics of Boolean algebras applied in functional analysis

We present some new constructions of Boolean algebras achieved by combinatorial methods which have applications in the theory of Banach spaces.

JORDI LOPEZ-ABAD, Université Paris 7

Partial unconditionality and Barriers

The purpose of this talk is to present a framework for studying weakly-null sequence of Banach spaces using the Ramsey property of the families of finite sets of integers called Barriers, introduced by Nash-Williams. We focus on “partial unconditionality” properties of weakly-null sequences of Banach spaces. Inspired by a recent work of S. J. Dilworth, E. Odell, Th. Schlumprecht and A. Zsak, we give a general notion of partial unconditionality that covers most of the known cases, including the classical Elton’s near unconditionality, convex unconditionality or Schreier unconditionality, and some new ones.

The method reduces the problem to the understanding of mappings $\varphi: \mathcal{B} \rightarrow \text{FIN} \times c_0$, where FIN denotes the family of finite sets, $\mathcal{B} \subseteq \text{FIN}$ is a barrier, and c_0 is the Banach space of sequences of real numbers converging to zero. We present several combinatorial results concerning these mappings, starting with the simpler mappings $\varphi: \mathcal{B} \rightarrow \text{FIN}$. One of the main results here is that every mapping $\varphi: \mathcal{B} \rightarrow c_0$ has a restriction which is, up to perturbation, what we call a L-mapping, *i.e.*, φ has a precise Lipschitz property and satisfies that the support $\text{supp } \varphi(s)$ of $\varphi(s)$ is included in s for every $s \in \mathcal{B}$. L-mappings allow to define naturally a weakly-null sequence, called L-sequence, associated to them.

Finally our approach shows that if for some notion of unconditionality \mathfrak{F} there is a weakly-null sequence with no \mathfrak{F} -unconditional subsequence, then there must be an L-sequence with no \mathfrak{F} -unconditional subsequence.

This is a joint work with S. Todorcevic.

JUSTIN MOORE, Boise State University, Boise ID, 83725-1555, USA

On linear orders with no real or Aronszajn suborders

In this talk we will consider what can be said about a linear order which contains neither real nor Aronszajn suborders. It is easily seen that σ -scattered orders fit this criterion. Baumgartner constructed an example which is not σ -scattered and contains neither a real nor Aronszajn suborder. Baumgartner’s example is necessarily not minimal with respect to not being σ -scattered. We have shown that PFA implies any minimal non- σ -scattered order of size \aleph_1 must either be a real or Aronszajn type. A version of this theorem for larger linear orders will also be discussed, along with the relevance to a theorem of Laver which asserts that the σ -scattered orders are well quasi-ordered. The work presented is joint work with Tetsuya Ishiu.

CRISTIAN ROSENDAL, University of Illinois at Urbana–Champaign

On the algebraic structure of the unitary group

We study the unitary group of separable infinite dimensional complex Hilbert space as a discrete group and show that all of its actions by isometries on a metric space have orbits of finite diameter. This property is enough to ensure that the unitary group also satisfies properties FH and FA of Serre.

This is a joint work with Eric Ricard of CNRS, Université Franche-Comté.

SLAWOMIR SOLECKI, University of Illinois, Department of Mathematics, 1409 W. Green St., Urbana, IL 61801, USA

A Fubini theorem

Let \mathcal{I}_0 be the σ -ideal of subsets of a Polish group generated by Borel sets which have perfectly many pairwise disjoint translates. I will present a Fubini-type theorem that holds between \mathcal{I}_0 and the σ -ideals of Haar measure zero sets and of meager sets. I will show how to use this result to give a simple proof of a generalization of a theorem of Balcerzak–Roslanowski–Shelah stating that \mathcal{I}_0 on $2^{\mathbb{N}}$ strongly violates the countable chain condition.

JURIS STEPRANS, York University

Localization properties of Silver forcing and countable support iterations

A partial order is said to have the n -localization property if every real added by the partial order can be approximated by an n -branching tree in the ground model. Well known arguments establishing this type of property for variants of Sacks forcing

do not generalize (in the obvious fashion) to the case of Silver forcing. It will be shown that, nevertheless, similar results do hold.

FRANKLIN TALL, University of Toronto, 40 St. George St., Toronto, ON M5S 2E4

PFA(S)[S]: mutually consistent consequences of PFA and $V = L$

Forcing with a coherent Souslin tree S , after iterating proper partial orders preserving S , produces interesting models. One gets “Souslin-type” consequences of PFA, as well as some combinatorics normally obtained from $V = L$. The combination of these two kinds of consequences leads to solutions of a number of previously intractable problems. The applications so far have been to topology, but one can expect to see other uses in the future. We shall discuss the method (pioneered by Todorcevic) and some applications.

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WILLIAM WEISS, University of Toronto

Problems caused by singular cardinals of countable cofinality

In ZFC, two recursive topological constructions are obstructed at singular cardinals of countable cofinality. We discuss recent work which tries to avoid this obstacle. The remaining problems are to exhibit a locally countable, countably compact space of large cardinality and a Bernstein set for any metric space.