Model Theory
Théorie des modèles
(Org: **Patrick Speissegger** (McMaster))

MATTHIAS ASCHENBRENNER, University of Illinois at Chicago
Asymptotic differential algebra

I will report on recent progress in the joint project with van den Dries and van der Hoeven mentioned in the title of the talk.

OZLEM BEYARSLAN, University of Illinois at Chicago
Pseudofinite Fields and Tournaments

A tournament on a set X is an irreflexive binary relation $R \subset X \times X$ such that, for every $x \neq y$ in X, exactly one of $R(x, y)$ and $R(y, x)$ holds. A pseudofinite field F interprets a tournament by the formula $\exists z : (x - y) = z^2$. The automorphism group of any field interpreting a 0-definable tournament can not have any involutions.

To generalize this observation, we will examine the effects of interpreting such structures on the automorphism groups of certain pseudofinite fields.

This is work in progress with Ehud Hrushovski.

GREGORY CHERLIN, Rutgers University
Connected Groups of Finite Morley Rank: Structure

We give a status report on the subject, with emphasis on recent achievements and challenges.

ALF DOLICH, McMaster University Hamilton, ON
Some Remarks on Weak O-minimality and Definable Completeness

We consider to what extent the role played by weakly o-minimal theories among all densely ordered theories is analogous to the role played by o-minimal theories among the definably complete densely ordered theories. In particular we ask whether certain results indicating that any sufficiently well-behaved definably complete theory is o-minimal or close to o-minimal may be generalized by dropping the assumption of definable completeness and weakening the conclusion from "o-minimal or close to o-minimal" to "weakly o-minimal or close to weakly o-minimal". We show through example that this is not the case.

DRAGOS GHIOCA, Department of Mathematics, McMaster University, 1280 Main Street West, Hamilton, Ontario L8S 2N5
A generalization of the Manin–Mumford Theorem

Let G be a semiabelian variety defined over a number field K. Let X be a subvariety of G defined over K^{alg}. The Manin–Mumford Theorem describes the intersection of $X(K^{\mathrm{alg}})$ with the torsion subgroup G_{tor} of G. More precisely, if X is an irreducible subvariety and $X(K^{\mathrm{alg}}) \cap G_{\mathrm{tor}}$ is Zariski dense in X, then X is a translate of an algebraic subgroup of G by a torsion point. In the present talk we show that we obtain the same conclusion about X assuming only that it contains a Zariski dense set of points of small height. Because all torsion points of G have height 0, we obtain that Manin–Mumford Theorem is a particular case of the result we present. Finally, we present a positive characteristic version of the Manin–Mumford Theorem in the context of Drinfeld modules.

DEIDRE HASKELL, McMaster University, 1280 Main St. W, Hamilton, ON L8P 2T4
Integral-definite rational functions in theories of valued fields

An analogue for valued fields of Hilbert's Seventeenth Problem asks for an algebraic characterisation of rational functions which map a definable subset of the field into the valuation ring. In this talk, I will describe a model-theoretic solution to this problem which is uniform across different theories of valued fields. I will apply it to algebraically closed valued fields, real closed valued fields, and model complete theories of difference and differential fields with a valuation.

TOBIAS KAISER, University of Regensburg, D-93040 Regensburg
An o-minimal version of the Riemann Mapping Theorem

We show that the germ of a Riemann map (*i.e.*, a biholomorphic map from a simply connected domain in the complex plane onto the unit ball) at an analytic corner of angle greater than 0 can be realized in a certain quasianalytic class, used by Ilyashenko in his solution of Hilbert's 16th problem. With this we are able to show that the Riemann map from a simply connected domain which is semianalytic and bounded, is definable in an o-minimal structure under some condition on the singularities of the domain.

SALMA KUHLMANN, University of Saskatchewan, Department of Mathematics and Statistics, McLean Hall, 106 Wiggins Road, Saskatoon, SK S7N 5E6
Integer Parts and Complements to Valuation Rings of Ordered Fields

An **integer part** (IP for short) Z of an ordered field K is a discretely ordered subring, with 1 as the least positive element, and such that for every $x \in K$, there is a $z \in Z$ such that $z \leq x < z + 1$. Mourgues and Ressayre establish the existence of an IP for any real closed field K by showing that there is an order preserving embedding φ of K into the field of generalized power series $k((G))$ such that $\varphi(K)$ is a truncation closed subfield (here k is the residue field and G the value group of K). An IP of K obtained in this way (*i.e.*, from a truncation closed embedding) is called a **truncation integer part** of K. IPs appear naturally in model theoretic arithmetic, algebra and analysis; *e.g.* Shepherdson showed that IPs of real closed fields are precisely the models of a fragment of Peano Arithmetic called Open Induction, whereas truncation IPs played a crucial role in Ressayre's investigations of the model theory of the real exponential field. In this talk, we analyze IPs from a valuation theoretic viewpoint and summarize their main special features. We investigate their connection to special (additive) complements of valuation rings of ordered fields. This approach reveals new interesting valuation theoretic properties of arbitrary *valued* fields (not just ordered fields); depending on whether such special complements exist. We discuss these properties and their implications, thereby giving an intrinsic valuation theoretic interpretation of truncation closed embeddings in fields of power series.

Joint work with F.-V. Kuhlmann and A. Fornasiero.

DAVID LIPPEL, Notre Dame, USA

CHRIS MILLER, Ohio State, Columbus
A proper reduct of the real projective hierarchy that defines sets in each projective level

There exist closed $E \subseteq \mathbb{R}$ such that $(\mathbb{R}, +, \cdot, E)$ defines a Borel isomorph of $(\mathbb{R}, +, \cdot, \mathbb{N})$, and so defines sets of every projective level, yet does not define $\mathbb{N}$, even when $(\mathbb{R}, +, \cdot, E)$ is further expanded by all subsets of every cartesian power of E.

Joint work with Harvey Friedman.

PAUL POTGIETER, University of South Africa, PO Box 392, Preller Street, Pretoria, South Africa, 0003
Nonstandard analysis, Hausdorff dimension and Brownian motion

In this paper we explore a nonstandard formulation of Hausdorff dimension. By considering an adapted form of the counting measure formulation of Lebesgue measure, we prove a nonstandard version of Frostman's lemma and find that Hausdorff dimension can be computed through a counting argument rather than by taking the infimum of a sum of certain covers. This formulation is then applied to obtain a simple proof of the doubling of the dimension of certain sets under a Brownian motion. In addition, the fractal properties of the rapid points of Brownian motion are explored using the new method, strengthening a result of Orey and Taylor's.

FERNANDO SANZ SÁNCHEZ, Universidad de Valladolid, Departamento de Álgebra, Geometría y Topología, Facultad de Ciencias, E-47005 VALLADOLID, Spain
Non-oscillating solutions of a differential equation and Hardy fields

Let $\varphi\colon x \mapsto \varphi(x)$, $x > a$ be a solution at infinity of an algebraic differential equation of order n, $P(x, y, y', \ldots, y^{(n)}) = 0$. We establish a geometric criterion so that the germ at infinity of φ, together with that of the identity function on $\mathbb{R}$, belongs to a common Hardy field.

More precisely, under the hypothesis that $\partial P/\partial y^{(n)}\big(x, \varphi(x), \varphi'(x), \ldots, \varphi^{(n)}(x)\big)$ is never zero, the criterion is the following non-oscillating property: for any polynomial $Q \in \mathbb{R}[x, y, y', \ldots, y^{(n-2)}]$, the function $x \mapsto Q\big(x, \varphi(x), \varphi'(x), \ldots, \varphi^{(n-2)}(x)\big)$ has a definite sign for $x \gg 0$. Immediate applications for differential equations of order one or two are given.

CAROL WOOD, Wesleyan University, Middletown, Connecticut, USA
Partial Differential Fields and Separation of Variables

A result of Johnson, Reinhart and Rubel (1995) shows that, unlike the one variable case, it is not possible in general to approximate solutions to partial differential equations via finite transcendence extensions. We indicate the relationship of this phenomenon to types in the model theory of partial differential fields in characteristic zero.