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Non-oscillating solutions of a differential equation and Hardy fields

Let $\varphi\colon x \mapsto \varphi(x)$, $x > a$ be a solution at infinity of an algebraic differential equation of order n, $P(x, y, y', \ldots, y^{(n)}) = 0$. We establish a geometric criterion so that the germ at infinity of φ, together with that of the identity function on $\mathbb{R}$, belongs to a common Hardy field.

More precisely, under the hypothesis that $\partial P/\partial y^{(n)}\big(x, \varphi(x), \varphi'(x), \ldots, \varphi^{(n)}(x)\big)$ is never zero, the criterion is the following non-oscillating property: for any polynomial $Q \in \mathbb{R}[x, y, y', \ldots, y^{(n-2)}]$, the function $x \mapsto Q\big(x, \varphi(x), \varphi'(x), \ldots, \varphi^{(n-2)}(x)\big)$ has a definite sign for $x \gg 0$. Immediate applications for differential equations of order one or two are given.