Mathematical Finance
Finance mathématique
(Org: **Len Bos** and/et **Anatoliy Swishchuk** (Calgary))

ABEL CADENILLAS, University of Alberta, Dept. of Mathematical and Statistical Sciences, Edmonton, Alberta T6G 2G1
Optimal Risk-Sharing with Effort and Project Choice

We consider first-best risk-sharing problems in which "the agent" can control both the drift (effort choice) and the volatility of the underlying process (project selection). In a model of delegated portfolio management, it is optimal to compensate the manager with an option-type payoff, where the functional form of the option is obtained as a solution to an ordinary differential equation. In the general case, the optimal contract is a fixed point of a functional that connects the agent's and the principal's maximization problems. We apply martingale/duality methods.

Joint work with Jaksa Cvitanic and Fernando Zapatero.

JOE CAMPOLIETI, Wilfrid Laurier University
New Families of Integrable Diffusion Models and their Applications to Finance

We present some recent developments in the construction and classification of new families of analytically solvable one-dimensional diffusion models for which transition densities and other quantities that are fundamental to financial modeling and derivatives pricing are represented in closed form. Our approach allows us to uncover new multi-parameter processes that are mapped into various simpler diffusions. From an asymptotic analysis of the boundary behaviour of the processes, we arrive at a rigorous characterization of the newly constructed diffusions with respect to probability conservation and the martingale property. Specifically, we analyze in detail three subfamilies of models arising from the underlying squared Bessel process (Bessel family), the CIR process (confluent hypergeometric family) and the Jacobi diffusion (hypergeometric family). We show that the Bessel family is a superset of the constant elasticity of variance (CEV) model. The former, in turn, is generalized by the confluent hypergeometric family. For these two families we find further subfamilies of conservative strict super-martingales and absorbed martingales. For the new classes of absorbed diffusions we also derive analytically exact first-passage time densities, as well as probability densities for the extrema of the processes. Formulas are reduced to integral representations or eigenfunction expansions involving special functions. New closed-form pricing formulas for standard Europeans, barrier options and lookback options also follow. We conclude by discussing other mathematical finance applications and possible extensions of our models to include jumps and markov switching.

MATT DAVISON, University of Western Ontario, Department of Applied Math
Some Optimal Control Problems in Natural Gas Storage

Real options theory is used to derive nonlinear partial integro-differential equations for valuing and optimally operating natural gas storage facilities. The equations incorporate a class of spot price models which exhibit the same time-dependent, mean-reverting dynamics and price spikes observed in energy markets. The operational characteristics of real storage units, including working gas capacities, variable delivery and injection rates, and cycling limitations, are incorporated. The model is illustrated with a solved numerical example of a salt cavern storage facility to illustrate the similarity between storage facilities and financial straddles. Depending on the amount of stored gas the relative influence of the put and call components vary.

This is joint work with Matt Thompson and Henning Rasmussen.

JEAN-MARIE DUFOUR, Université de Montréal
Testing portfolio efficiency with an unobservable zero-beta rate and non-Gaussian distributions: a finite-sample identification-robust approach

We consider the problem of testing portfolio efficiency when the zero-beta rate is unknown [Black Capital Asset Pricing Model (BCAPM)]. It is well known that standard asymptotically justified tests and confidence intervals are quite unreliable in this setup. We point out that this feature is associated with the fact that the zero-beta rate may be interpreted as a structural parameter that may be weakly identified, leading to a breakdown of standard asymptotic procedures based on estimated standard errors. The available exact procedures for the BCAPM, however, rely heavily on the assumption that model disturbances follow a Gaussian distribution, which does not appear to be satisfied by many financial return series. We propose exact simulation-based procedures for testing mean-variance efficiency of the market portfolio and building confidence intervals for the unknown zero-beta rate. The proposed methods are based on likelihood-ratio-type statistics, allow for a wide class of error distributions (possibly heavy-tailed) and are robust to weak identification of the zero-beta rate. Further, we suggest a general method which yields tighter bounds in both Gaussian and non-Gaussian cases. In order to build confidence intervals for the zero-beta rate in finite samples, a technique based on generalizations of the classic Fieller method (for the ratio of two parameters) is proposed. Empirical results on NYSE returns show that exact confidence sets are very different from the asymptotic ones, and allowing for non-Gaussian distributions substantially decreases the number of efficiency rejections.

ROBERT ELLIOTT, University of Calgary, Haskayne School of Business, 2500 University Drive NW, Calgary, AB T2N 1N4
New Results for Fractional Brownian Motion

Mathematically Fractional Brownian Motion is a difficult process to define. We shall present a new approach to Fractional Brownian Motion extending recent work of Hu, Oksendal, Duncan and Pasik Duncan to include processes with Hurst parameters $0 < H < 1$. A central tool for fractional Brownian motion is the analog of the Ito formula. We give new proofs and results. We also answer some of the criticisms of the use of fractional Brownian motion in financial modeling.

This is joint work with John van der Hoek of the University of Adelaide, Australia.

PETER FORSYTH, University of Waterloo, 200 University Avenue, Waterloo, Ontario
Dynamic Hedging Under Jump Diffusion with Transaction Costs

It is well known that the standard model of asset price processes, Geometric Brownian Motion, is not capable of reproducing the fat tails in observed price distributions. From a risk management point of view, the most troubling aspect of commonly used models is their inability to provide a useful hedging strategy in the presence of jumps.

If the price process follows a jump diffusion, it is known that a perfect hedge is not possible with a finite number of hedging instruments. It is also conventional wisdom that hedging with options is too expensive, due to the large transaction costs typical of the option market.

In this study, we suggest a dynamic hedging strategy based on hedging with the underlying and liquid options. We solve an optimization problem at each hedge rebalance date. We minimize both the "jump risk" and the transaction cost.

Simulation studies of this strategy, using typical market bid-ask spread data for the options, shows that using the underlying and options in the hedge portfolio does an excellent job of minimizing jump risk, as well as being not too costly in terms of total cumulative transaction costs.

This is joint work with Shannon Kennedy and Ken Vetzal.

ULRICH HORST, University of British Columbia, Department of Mathematics
Climate Risk, Securitization, and Equilibrium Bond Pricing

We propose a method of pricing financial securities written on non-tradable underlyings such as temperature or precipitation levels. To this end, we analyze a financial market where agents are exposed to financial and non-financial risk factors. The agents hedge their financial risk in the stock market and trade a risk bond issued by an insurance company. From the issuer's point of view, the bond's primary purpose is to shift insurance risks related to non-catastrophic weather events to financial markets. As such its terminal payoff and yield curve depend on an underlying climate or temperature process whose dynamics is independent of the randomness driving stock prices. We prove that if the bond's payoff function is monotone in the external risk process, it can be priced by an equilibrium approach. The equilibrium market price of climate risk and the equilibrium price process are characterized as solution of non-linear backward stochastic differential equations. Transferring the BSDEs into PDEs, we represent the bond prices as smooth functions of the underlying risk factors.

The talk is based on joint work with Matthias Muller.

ADAM KOLKIEWICZ, Department of Statistics and Actuarial Science, University of Waterloo, Waterloo, Ontario
Inverse First-Passage Problem with Applications to Credit Risk Modeling

In the paper we consider the inverse first-passage problem for a Brownian motion, which arises in the context of calibration of some models for equity and credit markets. The objective of the problem is to determine a deterministic barrier such that the corresponding exit time of the process has a given distribution. In contrast to the classical exit time problem, the inverse problem is less studied and even for the standard Brownian motion the existing results are not fully satisfactory. In the paper we identify shortcomings of existing techniques and present a method that addresses these issues.

ALI LAZRAK, University of British Columbia
Non-commitment in continuous time

This paper characterizes differentiable subgame perfect equilibria in a continuous time intertemporal decision optimization problem with non-constant discounting. The equilibrium equation takes two different forms, one of which is reminiscent of the classical Hamilton–Jacobi–Bellman equation of optimal control, but with a non-local term. We give a local existence result, and several examples in the consumption saving problem. The analysis is then applied to suggest that non-constant discount rates generate an indeterminacy of the steady state in the Ramsey growth model. Despite its indeterminacy, the steady state level is robust to small deviations from constant discount rates.

CRISTIANE LEMIEUX, University of Calgary, 2500 University Drive NW, Calgary, AB T2N 1N4
Quasi-Monte Carlo Methods and Finance

Quasi-Monte Carlo methods use highly-uniform point sets rather than random sampling in an attempt to improve upon the Monte Carlo method. These methods have found applications in different areas, most notably in finance. In this talk, I will introduce the general principles underlying quasi-Monte Carlo methods, and discuss different aspects of the interplay between these methods and financial problems. I will conclude with a brief overview of quasi-Monte Carlo constructions tailored to financial applications.

ALEX MELNIKOV, University of Alberta, Department of Math. and Stat. Sciences, Edmonton, AB
Partial Hedging for Valuation of Finance-Insurance Contracts

The talk is devoted to hedging methods developed in the modern financial mathematics and their applications to equity-linked life insurance. We study mixed finance-insurance contracts with fixed and flexible guarantees conditioned by survival status of the insured. In our setting, these instruments are based on two risky assets of a diffusion or jump-diffusion market. The first asset is responsible for the maximal size of a future profit while the second, more reliable, asset provides a flexible guarantee for the insured. The insurance company is considered as a hedger of a maximum of these assets conditioned by remaining lifetime of a client. The main attention is paid to quantile hedging, which, together with Black–Scholes (fixed guarantee) and Margrabe (flexible guarantee) formulae, creates effective actuarial analysis of such contracts. Some connections and further developments in mortality modeling and risk measures are discussed. Finally, we give numerical examples based on financial indices to demonstrate how our results can be applied to actuarial risk-management.

LUIS SECO, University of Toronto, Department of Mathematics, 40 St. George Street, Toronto, Ontario M5S 3G3
Collateralized Fund Obligations

Collateralized fund obligations provide an inexpensive way to finance leveraged fund investments. They have been quite popular in the low rate environment of the last few years. As credit derivatives, their pricing poses interesting challenges which can be studied with traditional methods. This talk will survey the product and will present a number of considerations involved in their pricing and risk exposures.

ANATOLIY SWISHCHUK, University of Calgary, 2500 University Drive NW, Calgary, AB T2N 1N4
Change of Time Method in Mathematical Finance

In this talk, we show how the change-of-time method works for different kind of models and problems arising in financial mathematics. We study the following three models in mathematical finance: geometrical Brownian motion model for stock price, mean-reverting model for commodity asset price and stochastic volatility model (that follows Cox–Ingersoll–Ross process) for Heston model of stock price. We apply the change-of-time method to derive (yet one more) the well-known Black–Scholes formula for European call option and to derive the explicit option pricing formula for European call option on mean-reverting model of commodity asset. We also derive the explicit formulas for variance and volatility swaps for financial markets with stochastic volatility following Cox–Ingersoll–Ross (1985) process (Heston (1993) model of stock price). Two numerical examples on the S&P60 Canada Index (January 1997–February 2002) to price variance and volatility swaps for Heston model and on AECO Natural Gas Index (1 May 1998–30 April 1999) to price European call option for mean-reverting asset model will be presented.

TONY WARE, University of Calgary, 2500 University Drive NW, Calgary, AB T2N 1N4
Swing options with continuous exercise

Swing options with continuous exercise can be used to model gas-storage contacts; options with discrete exercise opportunities and American options, can be viewed as limiting cases. The value of such an option may be found by solving a semilinear PDE, and I will discuss the numerical solution of such equations.