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Curious cubes and self-similar sums of squares

I will take mild issue with Hardy's dismissive remark:

"There are just four numbers (after 1) which are the sums of the cubes of their digits, viz. $153 = 1^3 + 5^3 + 3^3$, $370 = 3^3 + 7^3 + 0^3$, $371 = 3^3 + 7^3 + 1^3$, and $407 = 4^3 + 0^3 + 7^3$. This is an odd fact, very suitable for puzzle columns and likely to amuse amateurs, but there is nothing in it which appeals much to a mathematician. The proof is neither difficult nor interesting—merely a little tiresome. The theorem is not serious; and it is plain that one reason (though perhaps not the most important) is the extreme speciality of both the enunciation and the proof, which is not capable of any significant generalization."

In retaliation I nominate $1^3 + 5^3 + 3^3 = 153$, $16^3 + 50^3 + 33^3 = 165033$, $166^3 + 500^3 + 333^3 = 166500333$, $1666^3 + 5000^3 + 3333^3 = 16665000333$, ..., and turning to squares, $12^2 + 33^2 = 1233$, $88^2 + 33^2 = 8833$, Of course that last pair of examples is mathematically far more interesting, and I concentrate on its generalisation by reporting on work done some years ago jointly with Kurt Thomsen and Mark Wiebe, at the time undergraduate students at the University of Manitoba.