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Log-discrepancy and chromatic number of hyperplane arrangements

To *sign* an arrangement of affine hyperplanes in R^d is to designate, for each hyperplane, one of its two sides as being “positive”. A vertex is any intersection of d affinely independent hyperplanes. We seek a signing for which each vertex lies in the positive side of about 50% of the hyperplanes not containing the vertex. This differs from “discrepancy theory”, where the goal is to minimize the *difference* rather than the *ratio* between the number of positive and negative sides containing each vertex. Hence the phrase “log-discrepancy” in the title.

An old formula of Minty expresses the (circular) chromatic number of a graph in terms of finding a signing of low log-discrepancy in an associated complex. This motivates the invariant we study here. It also allows us to define a “chromatic number” for any hyperplane arrangement and, more generally, for any oriented matroid.

Using a probabilistic argument, we prove that the chromatic number of any loopless oriented matroid is bounded above by a function of its corank. This generalizes the observation that the chromatic number of a loopless connected graph (V, E) is bounded above (approximately) by the square root of its Betti number $|E| - |V| + 1$.

This is joint work with P. Hlineny and W. Hochstaettler.