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The combinatorics of sums of squares as studied via topology

Historically, the well-known identity

$$(x_1^2 + x_2^2)(y_1^2 + y_2^2) = (x_1y_1 - x_2y_2)^2 + (x_1y_2 + x_2y_1)^2$$

had led to a "sums of square" problem, namely to determine all identities of the type

$$(x_1^2 + \cdots + x_r^2)(y_1^2 + \cdots + y_s^2) = \phi_1^2 + \cdots + \phi_n^2$$

where $\phi_1, \ldots, \phi_n$ belong to a polynomial ring $\Lambda[x_1, \ldots, x_r\,;\, y_1, \ldots, y_s]$. In this generality the problem remains wide open. For the case when the commutative ring Λ is $\mathbf{Z}$, it reduces to a question of discrete mathematics, involving "signed intercalate matrices". In this talk I shall explain what intercalate matrices are, and show how ideas from algebraic topology can be used in the study of the combinatorics of such matrices.