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The geometry of Schur's theorem

One of the first results in additive combinatorics belongs to I. Schur and dates back to 1916. His motivation was to study "the local version" of the famous equation of Fermat: $x^n + y^n = z^n$. If there are integers x, y, z satisfying the above equation, then for every prime p, they also solve the congruence equation: $x^n + y^n \equiv z^n \pmod{p}$. He showed that the congruence equation has a non-trivial solution for all large primes p. This result follows from the next result, known as Schur's Theorem: *Let $r > 1$. Then there is a natural number $S(r)$, such as if $N > S(r)$ and if the numbers $\{1, 2, \dots, N\}$ are coloured with r colours, then there are three of them x, y, z of the same colour satisfying the equation: $x + y = z$.* We will give a geometric-combinatorial proof for Schur's Theorem and extend it, to prove a density version, similar to Roth's theorem for 3-term arithmetic progressions.